Math 302.102 Midterm #1 – October 22, 2010

This exam has 5 problems on 5 numbered pages and is worth a total of 50 points.

You have 50 minutes to complete this exam. Please read all instructions carefully, and check your answers. Show all work neatly and in order, and clearly indicate your final answers. Answers must be justified whenever possible in order to earn full credit. ***Unless otherwise specified, no credit will be given for unsupported answers, even if your final answer is correct.*** *Points will be deducted for incoherent, incorrect, and/or irrelevant statements.*

This exam is closed-book, although a handheld calculator is permitted. No other aids are allowed.

You must answer all of the questions in the space provided. Note that blank space is NOT an indication of a question's difficulty.

Instructor: Michael Kozdron Name: ________________________________

Problem	Score
1	
2	
3	
4	
5	

TOTAL: ____________

1. (*10 points*) Suppose that

$$f(x) = \begin{cases} cx^3, & \text{if } 0 < x < 2, \\ 0, & \text{otherwise.} \end{cases}$$

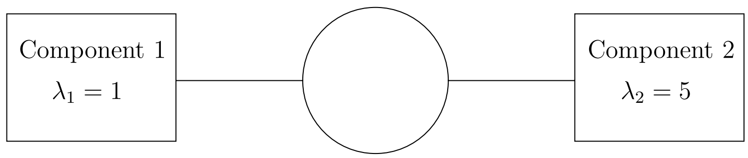
(a) Determine the unique value of c such that f is a legitimate probability density function.

For the remaining parts of this problem, suppose that X is a random variable having density f using the value of c you found in **(a)**.

(b) Determine $F(x)$, the distribution function of X.

(c) Determine $\mathbf{P}\{X > 1\}$.

2. (*10 points*) A mechanical system consists of two components. The components each perform independently, although their lifetimes, measured in days, do not have the same distribution. In fact, component i has an exponential distribution with parameter λ_i as shown in the following diagram.



The mechanical system will operate provided at least one component is performing. Determine the probability that the mechanical system will operate for at least 3 days.

Hint: Recall that if X is exponentially distributed with parameter $\lambda > 0$, then the density function of X is

$$f(x) = \begin{cases} \lambda e^{-\lambda x}, & \text{if } x \geq 0, \\ 0, & \text{if } x < 0. \end{cases}$$

3. (*10 points*)

(a) Suppose that A and B are *disjoint* events. It is known that $\mathbf{P}\{A\} = 0.4$ and $\mathbf{P}\{A \cup B\} = 0.8$. Determine $\mathbf{P}\{B\}$.

(b) Suppose that A and B are *independent* events. It is known that $\mathbf{P}\{A\} = 0.4$ and $\mathbf{P}\{A \cup B\} = 0.8$. Determine $\mathbf{P}\{B\}$.

(c) Suppose that A and B are events. It is known that $\mathbf{P}\{A\} = 0.4$, $\mathbf{P}\{A \cup B\} = 0.8$, and $\mathbf{P}\{B|A\} = 0.3$. Determine $\mathbf{P}\{B\}$.

4. (*10 points*) A basket contains 4 white balls, 5 red balls, and 6 black balls. A ball is selected at random and its colour is noted. The selected ball is then replaced along with 3 more balls of the same colour (so that there are now 18 balls in the basket). Then another ball is drawn at random from the basket.

(a) Determine the probability that the second ball drawn is red. *Hint*: You may find a tree diagram helpful here.

(b) Suppose that the second ball drawn is red. What is the probability that the first ball drawn was black?

5. (*10 points*) It is well-known that commercial airlines oversell seats on their flights since, for various reasons, passengers routinely do not show up. Suppose that historically 6% of Air Canada passengers do not show up. If Air Canada sells 122 tickets for an airplane that has 120 seats, what is (an expression for) the probability that there will be a seat for every passenger who shows up? You may assume that each passenger behaves independently.

Hint: Let X denote the number of passengers who do not show up.