Math 302.102 Fall 2010
Class Exercises from November 26, 2010
Problem 1. For a certain section of pine forest, the number of $Y$ of diseased trees per acre has a Poisson distribution with mean $\lambda=10$. The diseased trees are sprayed with an insecticide at a cost of $\$ 3$ per tree, plus a fixed overhead cost for equipment rental of $\$ 50$. Letting $C$ denote the total spraying cost for a randomly selected acre, find the expected value and standard deviation of $C$.

Problem 2. The number of eggs laid on a tree leaf by an insect of a certain type is a Poisson random variable with parameter $\lambda$. However such a random variable can only be observed if it is positive, since if it is 0 then we cannot know that such an insect was on the leaf. If we let $Y$ denote the observed number of eggs, then

$$
\mathbf{P}\{Y=i\}=\mathbf{P}\{X=i \mid X>0\}
$$

where $X$ is Poisson with parameter $\lambda$. Find $\mathbb{E}(Y)$.
Problem 3. Jessica is a compulsive shoe shopper, and she buys pairs of shoes according to a Poisson process with a rate (or intensity) of $\lambda=1$ pair per week.
(a) How many pairs of shoes is Jessica expected to buy in one year? (Recall that there are 52 weeks in a year.)
(b) Suppose that Jessica bought 8 pairs of shoes during the four weeks of February 2010. What is the probability that she bought 3 of them during the first week of February 2010?

Problem 4. Earthquakes in Klopstockia occur according to a Poisson process. Suppose that the expected value of the time until the next earthquake there is 2 years.
(a) What is the probability that there will be exactly 4 earthquakes in the next 10 years?
(b) What is the conditional probability that there will be exactly one earthquake in the year 2011, given that there will be exactly two in the 5 years 2011 to 2015 ?

Problem 5. Each game you play is a win with probability $p$. You plan to play 5 games, but if you win the fifth game then you will keep on playing until you lose.
(a) Find the expected number of games that you play.
(b) Find the expected number of games that you lose.

