Math 302.102 Fall 2010 Class Exercises from November 26, 2010

Problem 1. For a certain section of pine forest, the number of Y of diseased trees per acre has a Poisson distribution with mean $\lambda = 10$. The diseased trees are sprayed with an insecticide at a cost of \$3 per tree, plus a fixed overhead cost for equipment rental of \$50. Letting C denote the total spraying cost for a randomly selected acre, find the expected value and standard deviation of C.

Problem 2. The number of eggs laid on a tree leaf by an insect of a certain type is a Poisson random variable with parameter λ . However such a random variable can only be observed if it is positive, since if it is 0 then we cannot know that such an insect was on the leaf. If we let Y denote the observed number of eggs, then

$$\mathbf{P}\{Y = i\} = \mathbf{P}\{X = i \,|\, X > 0\}$$

where X is Poisson with parameter λ . Find $\mathbb{E}(Y)$.

Problem 3. Jessica is a compulsive shoe shopper, and she buys pairs of shoes according to a Poisson process with a rate (or intensity) of $\lambda = 1$ pair per week.

- (a) How many pairs of shoes is Jessica expected to buy in one year? (Recall that there are 52 weeks in a year.)
- (b) Suppose that Jessica bought 8 pairs of shoes during the four weeks of February 2010. What is the probability that she bought 3 of them during the first week of February 2010?

Problem 4. Earthquakes in Klopstockia occur according to a Poisson process. Suppose that the expected value of the time until the next earthquake there is 2 years.

- (a) What is the probability that there will be exactly 4 earthquakes in the next 10 years?
- (b) What is the conditional probability that there will be exactly one earthquake in the year 2011, given that there will be exactly two in the 5 years 2011 to 2015?

Problem 5. Each game you play is a win with probability p. You plan to play 5 games, but if you win the fifth game then you will keep on playing until you lose.

- (a) Find the expected number of games that you play.
- (b) Find the expected number of games that you lose.