Math 302.102 Fall 2010
Some Old Math 302 Exams

There is always a danger when distributing old exams for a class that students will rely on them *solely* for their final exam preparations. The final exam represents a culmination of a semester's worth of teaching and is very heavily influenced by the particular details of a given instructor in a given semester. While all of the material is generally standard from semester-to-semester, the specific focus and emphasis are not. As such, certain topics may be stressed more or less from one final exam to the next. In fact, it is quite likely that certain topics are omitted entirely.

It is therefore vital that students plan to review all of their lecture notes, handouts, assignments, and midterms in preparation for the final. While old exams are a valuable source of practice problems when used in conjunction with other reviews, as stated already, they are not a substitute for such review.

Worth mentioning, too, is the fact that there are always slight variations in the notation that is used from one semester to the next. Although most notation is standardized, not all of it is. Textbook authors, like professors, have their own favourite ways of writing things. And since textbooks are frequently changing, professors are often forced to change notation to adapt. For instance, whether one write $\mathbf{P}\{A\}$ or $P(A)$ or $\mathbb{P}(A)$ for the probability of the event A is unimportant since they all mean the same thing.

Attached are several old exams. In all instances, the question wording has been repeated verbatim from the original. The two midterms from 2010 were given in the other section of Math 302 this semester. The April 1989 final exam is from before the current UBC course numbering system was introduced.

The Math Club also produces a booklet of old exams. If you happen to purchase one, I would like to note that Problem #8 from April 2002 is not appropriate for Math 302 this semester.

Math 302 Section 101– First Midterm – October 13, 2010

Note that AB is shorthand for $A \cap B$ and ABC is shorthand for $A \cap B \cap C$. That is, $AB = A \cap B$ and $ABC = A \cap B \cap C$.

1. (*10 points*) The probability that an apparently healthy individual from a certain population has Ross's Disease is 1/100. The screening test for Ross's Disease gives a positive result with probability 96/100 for any person who has the disease, and with probability 8/100 for any person who doesn't have the disease. An apparently healthy individual from the population is tested, and the test result is positive. What is the probability that this person has the disease?

2. (*12 points*) An urn contains three red balls (with numbers 1, 2, and 3), two green balls (with numbers 1 and 2), and one yellow ball (with number 1). Two balls are removed randomly, without replacement. Find

(a) The probability that the two balls have the same colour.

(b) The conditional probability that the two balls have the same number, given that they have different colours.

3. (*9 points*) A and B are events such that $P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{2}$. Find $P(A \cup B)$ under each of the following assumptions:

(a) A and B are mutually exclusive

(b) $P(A|B) = \frac{1}{4}$.

4. (*8 points*) If the letters of "MOOSOMIN" are arranged in random order, what is the probability that both M's come before the first O? *Hint:* we don't care where the S, I and N are.

5. (*11 points*) Suppose that we have three events A, B, and C, all with nonzero probabilities, that are pairwise independent (i.e. A and B are independent, A and C are independent, and B and C are independent), but the intersection of any two is a subset of the third (i.e. $AB \subset C$, $AC \subset B$, $BC \subset A$).

(a) Show that $P(A) = P(B) = P(C)$. *Hint:* express $P(ABC)$ in three different ways.

(b) Express $P(A \cup B \cup C)$ in terms of $P(A)$. *Hint:* inclusion-exclusion.

Math 302 Section 101 – Second Midterm – November 10, 2010

1. (*10 points*) Suppose that X is a random variable with density

$$f(x) = \begin{cases} 2x - x^2, & \text{if } 0 < x < 1 \\ cx, & \text{if } 1 < x < 2 \\ 0 & \text{otherwise} \end{cases}$$

where c is a constant.

(a) Find the value of c.

(b) Find the variance of X.

2. (*10 points*) There are n kernels of popcorn in a pan. The number of these kernels that pop is a random variable with expected value 63 and variance 6.3. What is the distribution of X? *Assume the kernels are independent. Note that you must find n.*

3. (*10 points*) An opinion poll takes a random sample of 1350 voters in a large city. Suppose that 40% of the population support Party A, while 0.2% of the population support Party B. Find numerical approximations (using different distributions) for

(a) the probability that more than 544 of the people in the sample support Party A

(b) the probability that at most 2 of the people in the sample support Party B.

4. (*10 points*) A self-proclaimed "mind reader" has set up the following demonstration of her ability. She has placed 5 cards on the table. You are to write down 3 of these cards while her back is turned. She then looks at the cards and makes 3 choices of her own. She claims that she will pick at least two of your choices. Assuming that she has no special talents, what is the probability that she succeeds?

5. (*10 points*) Earthquakes in Klopstockia occur according to a Poisson process. Suppose that the expected value of the time until the next earthquake there is 2 years.

(a) What is the probability that there will be exactly 4 earthquakes in the next 10 years?

(b) What is the conditional probability that there will be exactly one earthquake in the year 2011, given that there will be exactly two in the 5 years 2011 to 2015?

Math/Stat 302 – Midterm 1 – June 17, 1996

1. (*10 points*) Let A, B, and C be independent events with probabilities 0.4, 0.5 and 0.3, respectively.

(a) Find the probability that exactly two of A, B, and C occur.

(b) Let $D = A \cup B \cup C$. Find $\mathrm{SD}(1 + I_D)$ where I_D is the indicator of the event D. *Note that I_D is a random variable such that $I_D = 1$ if D occurs and $I_D = 0$ if D does not occur.*

2. (*10 points*) Five cards are dealt from a standard deck of 52 cards.

(a) Find the probability that all cards are of the same suit.

(b) Find the probability that there is at least one ace.

(c) Find the probability that there is exactly one ace.

(d) Find the expected number of aces.

3. (*10 points*) A box contains three white balls and four red balls. Two balls are drawn from the box at random without replacement.

(a) Find the probability that the second draw is a red ball.

(b) Given that at least one of the two draws is a red ball, what is the probability that the second draw is a red ball?

(c) If 100 balls are drawn from a box at random with replacement, what is the distribution of the number of red balls?

4. (*10 points*) Suppose that in a game you pay \$4 to roll a fair die and then gain the same number of dollars as your roll. That is, in each game, if X is the number of your roll, then your net gain is $X - 4$ dollars. Suppose you play this game 100 times.

(a) What is the probability that you will lose \$50?

(b) What is the probability that you will win?

(continued)

5. (*10 points*) You roll a fair die once and record the number by X. Then I roll the same die repeatedly until I obtain a number which is greater than or equal to your number X. Let Y denote the number of rolls that I have to make.

(a) Find $P(X = 1, Y = k)$ for all $k = 1, 2, 3, \ldots$.

(b) Find $P(X = 2, Y = k)$ for all $k = 1, 2, 3, \ldots$.

(c) Find the joint distribution table for X and Y.

(d) Find $E(Y)$. Hint: Find $\sum_{k=1}^{\infty} k \cdot P(X = i, Y = k)$ for each $i = 1, 2, 3, 4, 5, 6$ and then use

$$E(Y) = \sum_{k=1}^{\infty} kP(Y = k) = \sum_{k=1}^{\infty} k \sum_{i=1}^{6} P(X = i, Y = k) = \sum_{k=1}^{\infty} \sum_{i=1}^{6} kP(X = i, Y = k)$$

Math/Stat 302 – Midterm 2 – July 17, 1996

1. (*7 points*) Suppose that X has density $f(x) = cx^2$ if $0 < x < 1$, and $f(x) = 0$ otherwise, where c is a constant.

(a) Find the constant c.

(b) Find the cumulative distribution function for X and sketch its graph.

2. (*15 points*) Suppose that (X, Y) has a uniform distribution on the triangle

$$\{(x, y) : 0 < x < y < 1\}.$$

(a) Find $P(X < Y^2)$.

(b) Find the marginal density $f_X(x)$.

(c) Suppose that 100 points are independently chosen uniformly at random from the triangle. What is the probability that the average of their x-coordinate is greater than $\frac{7}{18}$?

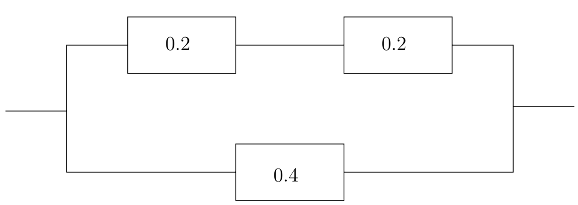
3. (*10 points*) Suppose that calls are arriving at an exchange according to a Poisson arrival process. Assume that the expected time that the third call arrives is 6 minutes.

(a) Find the probability that no calls arrive in the first minute.

(b) Find the probability that the third call arrives after 4 minutes.

4. (*10 points*) Suppose that X and Y are independent random variables such that X has a normal distribution with mean 3 and variance 2, and that Y has an exponential distribution with mean 1/2. Determine $E(X^2 e^Y)$.

5. (*8 points*) An electrical circuit consists of three components as in the following diagram. The lifetimes of the components, measured in days, have independent exponential distributions with death rates indicated in the diagram.



(a) Find the probability that the circuit will operate at least one day.

(b) What is the expected lifetime of the circuit?

The University of British Columbia
Sessional Examinations — April 1989

MATHEMATICS/STATISTICS 205

Time: 2 1/2 hours

INSTRUCTIONS: Calculators are allowed, no other aids. Show your work. This exam consists of 3 pages, including a table of the standard normal distribution on the last page.

[7] **1.** A husband and wife are going on vacation, and have a list of 10 possible destinations from which to choose. If they each choose 3 destinations at random, what is the probability that there will be exactly one destination that they both choose?

[10] **2.** A student takes a scholarship exam consisting of 10 questions, each worth 1 point. However, unless at least one of the first two questions is answered correctly, the markers will not look at the rest of the exam. On each question, she has probability 0.8 of getting the correct answer. The questions are independent.

(a) What is the probability that she will receive a mark of exactly 7 out of 10 (i.e. that she has exactly 7 questions right, *including* at least one of the first two)?

(b) Assuming that she receives a mark of 7 out of 10, what is the probability that *both* of the first two questions were answered correctly?

[16] **3.** A random variable Y has the density function

$$f_Y(y) = \begin{cases} \dfrac{1}{y^2} & \text{if } \dfrac{1}{2} < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

(a) Verify that this can be the density of a random variable.

(b) Find the cumulative distribution function $F_Y(y)$.

(c) Find the median of Y, i.e. a number c such that $P(Y \le c) = P(Y \ge c) = \dfrac{1}{2}$.

(d) Find the expected value $E(Y)$ and the variance $V(Y)$.

[16] **4.** Of the 30,000 inhabitants of a certain town, 300 believe the Earth is flat. You pick a random sample of 200 different people in this town.

(a) What is the expected number of people in your sample who believe the Earth is flat?

(b) Write an exact expression (but don't calculate it further) for the probability that fewer than 3 of the people in your sample believe the Earth is flat.

(c) What other distributions can be used to approximate the probability in (b)? Why are they applicable? What are the approximations? Calculate one approximation numerically.

[7] **5.** The time a student spends working on a certain exam problem has an exponential distribution with expected value 5 minutes. At 12:00, 10 students start working on this problem (independently, we hope!). What is the probability that they are all still working on it at 12:04?

[16] **6.** Two random variables Y_1 and Y_2 have the following joint probability distribution:

		Y_1		
		-1	0	2
Y_2	0	0.1	0.1	0.1
	1	0.1	0.2	0
	2	0	0.1	0.3

Find:

(a) $P(Y_1 Y_2 \leq 2)$.

(b) the conditional probability mass function of Y_2 given $Y_1 = 0$.

(c) the covariance of Y_1 and Y_2.

[10] **7.** The number of eggplants a grocery store sells on any given day is a random variable with expected value 10 and variance 32. Each day's sales are independent of the other days'. Let Y be the total number of eggplants sold in 200 days.

(a) What are the expected value and variance of Y?

(b) What distribution can be used to approximate the distribution of Y? What theorem makes this choice reasonable?

(c) Using this approximate distribution, find $P(1800 \leq Y \leq 1900)$.

[8] **8.** A certain radioactive sample emits an average of one gamma ray every 20 seconds. The numbers of gamma rays emitted during disjoint intervals of time are independent. What is the probability that at least two gamma rays will be emitted during a one minute interval?

[10] **9.** Two random variables X and Y have the joint density

$$f_{X,Y}(x,y) = \begin{cases} k\left(x + e^{2y}\right) & \text{if } 0 < x < 3 \text{ and } 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

where k is a constant.

(a) What is k?

(b) Are X and Y independent? Why?

(c) Find $P(X < Y)$.

[100] total

Area $\Phi(z)$ under the standard normal density curve to the left of z

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998
3.5	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998
3.6	.9998	.9998	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999