Math 302.102 Fall 2010
Assignment #7

This assignment is due at the beginning of class on Monday, November 22, 2010.

1. Let $0 < a < b$ be given positive constants, and suppose that the random vector (X, Y) has joint density function

$$f_{X,Y}(x,y) = \begin{cases} c(y-x), & \text{if } a < x < y < b, \\ 0, & \text{otherwise,} \end{cases}$$

where the value of the normalizing constant c is chosen so that $\displaystyle\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{X,Y}(x,y)\,\mathrm{d}x\,\mathrm{d}y = 1$.

(a) Determine the value of c. (Of course, your answer will depend on a and b.)

(b) Determine $f_Y(y)$, the marginal density function of Y, and $\mathbb{E}(Y)$, the expected value of Y.

(c) Determine $f_X(x)$, the marginal density function of X, and $\mathbb{E}(X)$, the expected value of X.

2. Suppose that the random vector (X, Y) is jointly distributed with joint density function

$$f_{X,Y}(x,y) = \begin{cases} 2e^{-x-y}, & \text{if } 0 < x < y < \infty, \\ 0, & \text{otherwise.} \end{cases}$$

(a) Determine the marginal density functions $f_X(x)$ and $f_Y(y)$.

(b) Based on your answer to **(a)**, are X and Y independent random variables? Justify your answer.

(c) Use the result of **(a)** to calculate $\mathbb{E}(X)$ and $\mathbb{E}(Y)$.