Math 302.102 Fall 2010
Assignment #4

This assignment is due at the beginning of class on Friday, October 15, 2010.

**1.** In each of the following cases, compute $\mathbf{P}\{0 < X < 2\}$ where the random variable $X$ has the given probability density function.

**(a)** $f(x) = \begin{cases} x^{-2}, & x \geq 1, \\ 0, & x < 1. \end{cases}$

**(b)** $f(x) = \begin{cases} 7e^{-7x}, & x \geq 0, \\ 0, & x < 0. \end{cases}$

**(c)** $f(x) = \begin{cases} \dfrac{e}{e^2-5}x^2e^{-x}, & -1 \leq x \leq 1, \\ 0, & \text{otherwise.} \end{cases}$

**(d)** $f(x) = \begin{cases} xe^{-x}, & x \geq 0, \\ 0, & x < 0. \end{cases}$

**2.** Suppose that the random variable $X$ has density function

$$f(x) = \begin{cases} \frac{1}{8}x & 0 \leq x \leq 4, \\ 0, & \text{otherwise.} \end{cases}$$

**(a)** Determine the value of $a$ such that $\mathbf{P}\{X \leq a\} = \dfrac{1}{2}$.

**(b)** Determine the value of $a$ such that $\mathbf{P}\{X \geq a\} = \dfrac{1}{4}$.

**3.** Suppose that the random variable $X$ has distribution function

$$F(x) = \begin{cases} 0, & x < 0, \\ \frac{1}{8}x^3, & 0 \leq x \leq 2, \\ 1 & x > 2. \end{cases}$$

**(a)** Compute $\mathbf{P}\{X \leq 1\}$.

**(b)** Compute $\mathbf{P}\{0.5 \leq X \leq 1.5\}$.

**(c)** Determine the value of $a$ such that $\mathbf{P}\{X \leq a\} = \dfrac{1}{2}$.

**(d)** Determine the value of $a$ such that $\mathbf{P}\{X \geq a\} = \dfrac{1}{4}$.

**4.** Suppose that $X$ is a normally distributed random variable with density function

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$$

for $-\infty < x < \infty$. Using a table of normal probabilities, compute each of the following probabilities accurate to 4 decimal places.

**(a)** $\mathbf{P}\{X > 1\}$.

**(b)** $\mathbf{P}\{X < 1\}$.

**(c)** $\mathbf{P}\{X \le 1\}$.

**(d)** $\mathbf{P}\{-1 \le X \le 1\}$.

**(e)** $\mathbf{P}\{X \le 2\}$.

**(f)** $\mathbf{P}\{X \ge -2\}$.

**(g)** $\mathbf{P}\{-2 \le X < 3\}$.

**(h)** $\mathbf{P}\{-1 \le X \le 3\}$.

**5.** Suppose that the lifetime $X$ (in years) of a particular television model is exponentially distributed with parameter $\lambda = 1/2$ so that the density function of $X$ is

$$f(x) = \begin{cases} \frac{1}{2} e^{-x/2}, & x \ge 0, \\ 0, & x < 0. \end{cases}$$

Suppose that three televisions of this particular model are selected at random, and assume that television lifetimes are independent.

**(a)** Determine the probability that all three televisions last for at least two years.

**(b)** Determine the probability that exactly one television lasts for less than one year (so that the other two last for at least one year).

**6.** Toyota vehicles have been scrutinized lately because of the surge in recalls that Toyota have announced. Suppose that cars account for 65% of Toyota's vehicle production, trucks account for 20% of Toyota's vehicle production, and vans account for the remaining 15% of their vehicle production. Suppose further that 10% of Toyota cars are recalled, 8% of Toyota trucks are recalled, and 12% of Toyota vans are recalled. Assuming that vehicles are recalled independently of other vehicles, what is the probability that a randomly selected recalled vehicle is a truck? *Note: In order to receive full points, you must answer this question by carefully defining events $A$, $B_1$, $B_2$, and $B_3$ and applying Bayes' Rule symbolically. You many use a tree diagram for motivation and intuition, but your written solution needs to be symbolic.*