Math 302.102 Fall 2010
Assignment #3

This assignment is due at the beginning of class on Wednesday, October 6, 2010.

1. It is known that in a certain population 5% of all men are colour blind and 0.25% of all women are colour blind. Suppose that one person is picked at random from this population and is found to be colour blind.

(a) Assuming that men and women each make up the same proportion of the population, what is the probability that this person is a man?

(b) If there are twice as many women as men in this population, what is the probability that this person is a man?

2. A manufacturing process produces integrated circuit chips. Over the long run, the fraction of bad chips produced by the process is 20%. Thoroughly testing a chip to determine whether it is good or bad is rather expensive, so a cheap chip test is tried. All good chips will pass the cheap chip test, but so will 10% of the bad chips.

(a) Given that a chip passes the cheap chip test, what is the probability that it is a good chip?

(b) If a company using this manufacturing process sells all chips which pass the cheap chip test, over the long run what percentage of chips sold will be bad?

Note: It is often the case in quality control that it is not possible to examine every manufactured item to determine whether or not it is up to standards. Testing might be rather expensive and so the manufacturers would want to limit the number of tests performed. Another reason why a sample might be needed is that the examination could destroy the item. For instance, to actually determine whether or not a package of Raisin Bran contains two scoops of raisins, one needs to physically open the package of Raisin Bran and check. Of course, since the package was opened it can no longer be sold. Other items that might be destroyed after quality control testing are spark plugs and coffee filters.

3. Suppose that a laboratory test on a blood sample of people from a certain population yields one of two results: positive or negative. It is found that 95% of people with a particular disease produce a positive result. But 2% of people without the disease will also produce a positive result (known as a *false positive*). Suppose that 1% of the population actually has the disease. What is the probability that a person chosen at random from the population will have the disease given that the person's blood yields a positive result?

4. (Continued from Problem #3) Suppose that a doctor is examining a patient from the population in the previous problem. This patient was NOT chosen at random. Rather, the patient walked into the doctor's office because he was feeling sick. After examining the patient, but not seeing the results of the blood test, the doctor's opinion is that there is a 30% chance that the patient has the disease. How should the doctor revise her opinion after seeing a positive blood test?

5. Consider the following simplified version of the casino game of craps. Player A is rolling a standard six-sided fair die. If the die shows 3 on the first roll, then Player A wins. If the die shows 1 or 6 on the first roll, then Player A loses. If the die shows 2, 4, or 5 on the first roll, then Player A repeatedly rolls the die until either a 3 appears in which case Player A loses, or the die shows the same number as on the first roll (either 2, 4, or 5) in which case Player A wins. Determine the probability that Player A loses.

6. For each of the following functions f, determine whether or not there is a value of c that makes f a legitimate probability density function. If such a c exists, compute its value. If such a c does not exist, carefully explain why.

(a) $f(x) = \begin{cases} cx^{-2}, & x \geq 1, \\ 0, & x < 1. \end{cases}$

(b) $f(x) = \begin{cases} cx^{-1}, & x \geq 1, \\ 0, & x < 1. \end{cases}$

(c) $f(x) = \begin{cases} cx^2e^{-x}, & -1 \leq x \leq 1, \\ 0, & \text{otherwise.} \end{cases}$

(d) $f(x) = \begin{cases} cx^2e^{-x}, & x \geq 0, \\ 0, & x < 0. \end{cases}$

(e) $f(x) = \begin{cases} cxe^{x}, & x \leq 0, \\ 0, & x > 0. \end{cases}$

(f) $f(x) = \begin{cases} cxe^{x}, & -1 \leq x \leq 1, \\ 0, & \text{otherwise.} \end{cases}$