Stat 862 (Winter 2007)
Assignment #2

This problem should be done with the help of a computer (say, MAPLE or MATLAB). It will illustrate the various long term behaviour of Markov chains.

For each of the following transition matrices, compute $\mathbf{P}^n$ for a sufficiently large power n and interpret the results. Does

$$\lim_{n\to\infty} \mathbf{P}^n$$

exist? Does

$$\lim_{n\to\infty} p_n(i,j) = \pi_j$$

independent of i?

(a)

$$\mathbf{P} = \begin{bmatrix} 3/4 & 0 & 1/4 \\ 1/8 & 1/8 & 3/4 \\ 1/12 & 1/4 & 2/3 \end{bmatrix}$$

(b)

$$\mathbf{P} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1/5 & 4/5 & 0 & 0 \\ 1/6 & 0 & 1/6 & 2/3 \\ 0 & 1/10 & 3/5 & 3/10 \end{bmatrix}$$

(c) Simple random walk on $S = \{0, 1, 2, 3, 4\}$ with reflecting boundaries.

$$\mathbf{P} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1/2 & 0 & 1/2 & 0 & 0 \\ 0 & 1/2 & 0 & 1/2 & 0 \\ 0 & 0 & 1/2 & 0 & 1/2 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

(d) Simple random walk on $S = \{0, 1, 2, 3, 4\}$ with absorbing boundaries.

$$\mathbf{P} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1/2 & 0 & 1/2 & 0 & 0 \\ 0 & 1/2 & 0 & 1/2 & 0 \\ 0 & 0 & 1/2 & 0 & 1/2 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

(e)

$$\mathbf{P} = \begin{bmatrix} 3/4 & 0 & 1/4 & 0 & 0 \\ 0 & 1/2 & 0 & 1/2 & 0 \\ 1/8 & 0 & 2/3 & 0 & 5/24 \\ 0 & 5/6 & 0 & 1/6 & 0 \\ 0 & 0 & 1/7 & 0 & 6/7 \end{bmatrix}$$