Stat 851 Winter 2008 Assignment #7

This assignment is due on Monday, March 24, 2008.

1. Let $\alpha > 0$ be fixed. Suppose that X_1, X_2, \ldots are independent random variables all defined on the probability space (Ω, \mathcal{A}, P) . Suppose further that

$$P\{X_n = 0\} = 1 - \frac{1}{n^{\alpha}}$$
 and $P\{X_n = n\} = \frac{1}{n^{\alpha}}$.

- (a) Show that $X_n \to 0$ in probability. (Note that this result holds even without the indpendence assumption.)
- (b) Show that $X_n \to 0$ completely if and only if $\alpha > 1$. (Conclude that $X_n \to 0$ almost surely if and only if $\alpha > 1$.)
- (c) Show that $X_n \to 0$ in L^p if and only if $\alpha > p$.
- 2. Complete the following exercises from pages 148–149:
 - #17.3, 17.4, 17.9, 17.10, 17.13
- **3.** Complete the following exercise from page 73:
 - #10.13