Stat 851 Winter 2008 Assignment #6

This assignment is due on Monday, March 3, 2008.

**1.** Consider the probability space  $(\Omega, \mathcal{A}, P)$  where  $\Omega = [0, 1]$ ,  $\mathcal{A} = \mathcal{B}([0, 1])$  are the Borel sets in [0, 1], and P is the uniform probability measure on [0, 1]. Let  $X : [0, 1] \to \mathbb{R}$  be the simple random variable

$$X(\omega) = 2 \cdot \mathbb{1}_{[0,\frac{1}{4})}(\omega) + 3 \cdot \mathbb{1}_{[\frac{1}{4},\frac{5}{8})}(\omega) + 7 \cdot \mathbb{1}_{[\frac{5}{8},\frac{3}{4})}(\omega) + 6 \cdot \mathbb{1}_{[\frac{3}{4},1]}(\omega).$$

- (a) Sketch the graph of  $\omega$  vs.  $X(\omega)$ .
- (b) Determine  $F_X$ , the distribution function of X.

(c) Compute the Riemann-Stieltjes integral 
$$\int_{-\infty}^{\infty} x^2 F_X(dx) = \int_{-\infty}^{\infty} x^2 dF_X(x).$$

- 2. Complete the following exercise from page 63:
  - #9.19
- 3. Complete the following exercises from pages 73–74:
  - #10.1, 10.6, 10.13

There are some typos in the answers for 10.6. In particular, the answers for (c) and (d) should be switched, and the answer for (e) needs to be multiplied by 2.