Stat 452 Fall 2011
Assignment #4

This assignment is due at the beginning of class on Monday, November 21, 2011.

1. Exercise 7.41 page 363

2. Exercise 7.44 page 363

3. Let $X_1, \ldots, X_n$ be iid random variables with density

$$f(x|\theta) = \frac{\theta}{(1+x)^{1+\theta}}, \quad x > 0,\ \theta > 0.$$

(a) Find the method of moments estimator of θ. What condition on θ is needed to make this method applicable in this case?

(b) Find the maximum likelihood estimator of $\frac{1}{\theta}$.

(c) Does there exist a sufficient statistic for θ other than the vector of order statistics? If so, find it.

(d) Find the Cramér-Rao bound for unbiased estimation of $\frac{1}{\theta}$.

(e) Does there exist an unbiased estimator of $\frac{1}{\theta}$ whose variance equals the Cramér-Rao bound for unbiased estimation of $\frac{1}{\theta}$? If so, find it.

4. Let $X_1, \ldots, X_n$ be iid random variables with the Poisson distribution

$$P_\theta(X_1 = x) = \frac{e^{-\theta}\theta^x}{x!}, \quad x \in \{0, 1, \ldots\},$$

where we know that $\theta \geq 1$.

(a) Find the maximum likelihood estimator $\hat{\theta}$ of θ; that is, the value of θ which maximizes the likelihood function under the restriction that $\theta \geq 1$.

(b) Is $\hat{\theta}$ an unbiased estimator of θ? Justify your answer.

5. Exercise 7.58 page 366

6. Exercise 7.59 page 366

7. Exercise 7.60 page 366