Stat 452 Fall 2011
Assignment #2

This assignment is due at the beginning of class on Wednesday, October 5, 2011.

1.   Let X be an observation from a $N(0, \sigma^2)$ population, $\sigma^2 > 0$. Is $|X|$ sufficient for σ^2? Justify your answer.

2.   Exercise 6.2 page 300

3.   Exercise 6.21 (c) page 302. Justify your answer.

4.   Exercise 6.9 (c) page 301

5.   Exercise 6.3 page 300

6.   Let $X_1, X_2, \ldots, X_n$ be iid with the geometric distribution

$$P_\theta(X = x) = \theta(1 - \theta)^{x-1}, \quad x = 1, 2, \ldots, \quad 0 < \theta < 1.$$

Show that

$$\sum_{i=1}^{n} X_i$$

is sufficient for θ and find the family of distributions of this statistic.

7.   Show that the family of gamma distributions with density given by

$$f(x|\theta) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}, \quad x > 0, \ \alpha > 0, \ \beta > 0$$

is an exponential family for the case where exactly one of the two parameters is known and the other unknown, as well as for the case where both parameters are unknown. For each of these three cases, find the natural parameter space of the family.

8.   Let $\Theta = \{\theta_1, \theta_2, \theta_3\}$. Consider the following family of distributions.

x	$P_{\theta_1}(X = x)$	$P_{\theta_2}(X = x)$	$P_{\theta_3}(X = x)$
1	0.45	0.24	0.15
2	0.30	0.16	0.10
3	0.15	0.08	0.05
4	0.10	0.52	0.70

Let T be defined by $T(1) = T(2) = T(3) = 5$ and $T(4) = 8$. Show that $T(X)$ is sufficient for θ.