

Stat 441 Winter 2009
Assignment #2

This assignment is due at the beginning of class on Wednesday, January 21, 2009.

1. Read Chapter 3, pages 21–31, of Higham.
 - Everything in this section *should* be familiar from STAT 251 and STAT 252.
 - Make sure you can do Exercises 3.1 through 3.9.

2. Read Chapters 5, 6, and 7, pages 45–72, of Higham.
 - Chapter 5 is reasonably straightforward.
 - Chapters 6 and 7 develop “geometric Brownian motion” as the model of the stock price movement. Read through these chapters to get a feel for what is to come.

3. The following exercises are from the printed lecture notes.
 - Exercises 5.5 and 5.7
 - Exercise 5.10 is important.
 - Exercises 6.5 and 6.6

4. Suppose that $\{S_n, n = 0, 1, 2, \dots\}$ is a simple random walk starting at 0, and consider the filtration $\{\mathcal{F}_n, n = 0, 1, 2, \dots\}$ given by $\mathcal{F}_n = \sigma(S_0, \dots, S_n)$. Compute $\mathbb{E}(\sin(S_{n+1})|\mathcal{F}_n)$. Is $\{\sin(S_n), n = 0, 1, 2, \dots\}$ a martingale. If not, find a function of $\sin(S_n)$ that is a martingale.

5. Suppose that $\{X_j, j = 0, 1, 2, \dots\}$ is a martingale with respect to the filtration $\{\mathcal{F}_n, n = 0, 1, 2, \dots\}$. Let $\{Y_j, j = 0, 1, 2, \dots\}$ be defined as

$$Y_j = f(X_0, \dots, X_j).$$

Define the process $\{M_n, n = 0, 1, 2, \dots\}$ by setting $M_0 = 0$ and

$$M_n = \sum_{j=1}^n Y_{j-1}(X_j - X_{j-1})$$

for $n = 1, 2, \dots$

- (a) Verify that $\{M_n, n = 0, 1, 2, \dots\}$ is a martingale with respect to the filtration $\{\mathcal{F}_n, n = 0, 1, 2, \dots\}$.
- (b) Compute $\mathbb{E}(M_n^2 - M_{n-1}^2 | \mathcal{F}_{n-1})$. *Hint: Use $(M_n - M_{n-1})^2$.*
- (c) Use the result of (b) to conclude that

$$\mathbb{E}(M_n^2 | \mathcal{F}_{n-1}) = \sum_{j=1}^n Y_{j-1}^2 \mathbb{E}[(X_j - X_{j-1})^2].$$

(continued)

(d) Use the tower property of conditional expectation to immediately deduce that

$$\mathbb{E}(M_n^2) = \sum_{j=1}^n \mathbb{E}(Y_{j-1}^2) \mathbb{E}[(X_j - X_{j-1})^2].$$

6. Complete the MATLAB exercises from Chapters 1, 2, 3, and 5. I'm not sure how straightforward P5.2 is.

7. Write MATLAB programs to simulate a simple random walk of 100 steps, 1000 steps, and 10000 steps.