Stat 352 Winter 2008
Assignment #5

This assignment is due at the beginning of class on Thursday, March 20, 2008, except for Problems 1.(a) and 1.(b) which are due at the beginning of class on Tuesday, March 18, 2008.

1. Suppose that the random variable Y has a Raleigh(θ) distribution where $\theta > 0$ is an unknown parameter so that

$$f(y|\theta) = \frac{2}{\theta} y e^{-y^2/\theta}, \quad y > 0.$$

Suppose further that the prior for θ is given by

$$g(\theta) = \frac{2}{\sqrt{\pi}} e^{-\theta^2}, \quad \theta > 0.$$

(a) Determine an expression for the posterior density function $f(\theta|y)$.

(b) Determine the posterior density function $f(\theta|y = 1)$ by numerically estimating the normalizing constant.

(c) Use R to simulate a random variable having the posterior density you found in (b). Use the prior as your enveloping function.

(d) Use R to simulate several hundred/thousand random variables having the posterior density you found in (b).

2. Researchers at the University of Saskatchewan Medical Center did a study of hereditary patterns in ice cream preference. They asked generations of families their favourite flavour of ice cream. They found that if someone preferred strawberry ice cream, their children had a probability of 0.5 of preferring strawberry ice cream, a probability of 0.1 of preferring chocolate ice cream, and a probability of 0.4 of preferring vanilla ice cream. If someone preferred chocolate ice cream, their children had a probability of 0.2 of preferring strawberry, a probablility of 0.3 of preferring chocolate, and a probability of 0.5 of preferring vanilla. If someone preferred vanilla, their children had a probability of 0.1 of preferring strawberry, a probability of 0.2 of preferring chocolate, and a probability of 0.7 of preferring vanilla.

(a) Write the transition matrix P for the above Markov chain. Is this a regular Markov chain?

(b) Find the equilibrium vector, and clearly show why it is the equilibrium vector.

(c) If Jay prefers chocolate ice cream, what is the probability that his grandchild will prefer vanilla ice cream?

3. Consider a Markov chain which has four states called A, B, C, and D. Let its transition matrix P be given below. (Let the states A, B, C, and D correspond to the 1st, 2nd, 3rd, and 4th rows and columns, respectively.)

$$P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0.2 & 0.6 & 0.2 \\ 0.1 & 0.1 & 0.1 & 0.7 \end{bmatrix}$$

If the Markov chain starts in state C, what is the probability that it will eventually end up in state A?

4. Consider a Markov chain with state space $\{0, 1, 2\}$ and transition matrix

$$P = \begin{bmatrix} 0.4 & 0.2 & 0.4 \\ 0.6 & 0 & 0.4 \\ 0.2 & 0.5 & 0.3 \end{bmatrix}.$$

What is the probability in the long run that the chain is in state 1? Solve this problem in two ways: (i) by raising the matrix to a high power, and (ii) by directly computing the invariant/stationary probability vector (as a left eigenvector).