Stat 352 Winter 2008
Assignment #2

This assignment is due at the beginning of class on Thursday, January 31, 2008.

1. Suppose that for each individual of a large population there is a pair of genes, each of which can be either x or X, that controls eye colour. Those with xx have blue eyes, while heterozygotes (those with xX or Xx) and those with XX have brown eyes. The proportion of blue-eyed individuals is p^2 and of heterozygotes is $2p(1-p)$, where $0 < p < 1$. Each parent transmits one of its own genes to the child. If a parent is a heterozygote, the probability that it transmits the gene of type X is 1/2.

(a) Assuming random mating, show that among brown-eyed children of brown-eyed parents, the expected proportion of heterozygotes is $2p/(1+2p)$.

(b) Suppose that Jessica, a brown-eyed child of brown-eyed parents, marries a heterozygote, and that they have n children, all brown-eyed. Find the posterior probability that Jessica is a heterozygote.

(c) Find the probability that Jessica's first grandchild has blue eyes.

2. Consider two coins, C_1 and C_2, with the following characteristics: $P\{\text{heads}|C_1\} = 0.6$ and $P\{\text{heads}|C_2\} = 0.4$. Choose one of the coins at random and imagine spinning it repeatedly. Given that the first two spins from the chosen coin are tails, what is the expected number of additional spins until a head shows up?

3. Suppose there is a $\beta(4,4)$ prior on the probability θ that a coin will yield a head when spun in a specified manner. The coin is independently spun 10 times, and heads appears fewer than 3 times. You are not told how many heads were seen, only that the number is (strictly) less than 3. Calculate your exact posterior density (up to a constant of proportionality) for θ.

4. Suppose that your prior distribution for θ, the proportion of Canadians who support the return of the death penalty, is beta with mean 0.6 and standard deviation 0.3.

(a) Determine the parameters a and b of your prior distribution.

(b) A random sample of $n = 1000$ Canadians is taken, and 35% support a return of the death penalty. Determine the posterior mean and posterior variance of θ based on this data.

5. Suppose that there are N buses in Regina sequentially numbered from 1 to N. You see a bus at random; it is numbered 203. You wish to estimate N. Assume your prior distribution on N is geometric with mean 100; that is,

$$g(N) = \frac{1}{100}\left(\frac{99}{100}\right)^{N-1}, \quad N = 1, 2, 3, \ldots.$$

(a) What is your posterior distribution for N?

(b) What are the posterior mean and standard deviation of N? (Either sum the infinite series analytically or approximate them on a computer.)