Stat 352 Winter 2008
Assignment #1

This assignment is due at the beginning of class on Tuesday, January 22, 2008.

1. Compute the approximate value of

$$\int_0^1 x^2 \, \mathrm{d}x$$

using a Riemann midpoint sum and four partitions of equal width.

2. Approximately 1/125 of all births are fraternal twins and 1/300 of births are identical twins. Elvis Presley had a twin brother (who died at birth). What is the probability that Elvis was an identical twin? (You may approximate the probability of a boy or a girl as 1/2.)

3. Suppose that $Y|\theta \sim \mathrm{Bin}(n, \theta)$ where θ follows a $\beta(a, b)$ prior density. That is, assume

$$g(\theta) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} \theta^{a-1}(1-\theta)^{b-1}, \quad 0 \le \theta \le 1.$$

Determine the posterior distribution of θ given $Y = y$.

4. Recall that a random variable Y is said to be Poisson with parameter θ if it has density function

$$f(y|\theta) = \frac{\theta^y e^{-\theta}}{y!}, \quad y = 0, 1, 2, \ldots.$$

Suppose that the random variables $Y_1, Y_2, \ldots, Y_n$ are i.i.d. Poisson(θ) where the parameter θ is unknown.

- **(a)** Compute $f(y_1, y_2, \ldots, y_n|\theta)$, the likelihood function. (In STAT 252, we wrote the likelihood function as $L(\theta)$.)
- **(b)** If the prior distribution of θ is $\Gamma(\alpha, \beta)$, determine the posterior distribution of θ given $\{Y_1 = y_1, Y_2 = y_2, \ldots, Y_n = y_n\}$.

5. A random sample of n University of Regina undergraduate students is drawn and each of their weights is measured. The average weight of the n sampled students is $\overline{y} = 150$ pounds. Assume that the weights of U of R undergraduates are normally distributed with unknown mean θ and known standard deviation 20 pounds. Suppose that your prior distribution for θ is normal with mean 180 pounds and standard deviation 40 pounds.

- **(a)** Determine the posterior distribution for θ. (Give your answer as a function of n.)
- **(b)** A new student is sampled at random from the same population and has a weight of $\tilde{y}$. Determine $f(\tilde{y}|y)$, the posterior predictive distribution for $\tilde{y}$. (Your answer will still depend on n.)