Statistics 351 Midterm #2 – November 17, 2006

This exam is worth 40 points.

There are 5 problems on 5 numbered pages. You may attempt all five and your four highest scores will be taken as your mark. You might want to read all five questions before you begin.

You have 50 minutes to complete this exam. Please read all instructions carefully, and check your answers. Show all work neatly and in order, and clearly indicate your final answers. Answers must be justified whenever possible in order to earn full credit. ***Unless otherwise specified, no credit will be given for unsupported answers, even if your final answer is correct.*** *Points will be deducted for incoherent, incorrect, and/or irrelevant statements.*

This exam is closed-book, except that one $8\frac{1}{2} \times 11$ *double-sided page of handwritten notes is permitted. No other aids are allowed.*

You must answer all of the questions in the space provided. Note that blank space is NOT an indication of a question's difficulty.

Name: ____________________________________

Instructor: Michael Kozdron

Problem	Score
1	
2	
3	
4	
5	

TOTAL: ____________

1. (*10 points*) Suppose that the random vector $\mathbf{X} = (X_1, X_2)'$ has a multivariate normal distribution with mean vector $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Lambda}$ given by

$$\boldsymbol{\mu} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad \text{and} \quad \boldsymbol{\Lambda} = \begin{pmatrix} 2 & -2 \\ -2 & 3 \end{pmatrix}.$$

(a) Prove that $\boldsymbol{\Lambda}$ is strictly positive definite. (That is, prove that $\boldsymbol{\Lambda}$ is positive definite but not non-negative definite.)

(b) Determine the characteristic function $\phi_{\mathbf{X}}(\mathbf{t})$ of $\mathbf{X}$.

(c) Let $Y_1 = X_1 - 2X_2$ and $Y_2 = X_1 + X_2$. Determine the distribution of the random vector $\mathbf{Y} = (Y_1, Y_2)'$.

2. (*10 points*) If the random vector $\mathbf{X} = (X_1, X_2)'$ has the multivariate normal distribution

$$\mathbf{X} \in N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix}\right)$$

prove that the random variables X_1 and $X_1 + X_2$ are independent.

3. (*10 points*) Suppose that X_1, X_2, X_3, X_4 are independent $U(0,1)$ random variables. Determine the density function for $X_{(3)} - X_{(2)}$.

4. (*10 points*) If the random vector $\mathbf{X} = (X_1, X_2)'$ has the multivariate normal distribution

$$\mathbf{X} \in N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right)$$

compute $E(\max\{X_1, X_2\})$.

Hint: In order to compute the resulting double integral, you might consider switching the order of integration.

5. (*10 points*) Suppose that the random vector $\mathbf{X} = (X_1, X_2)'$ has the multivariate normal distribution

$$\mathbf{X} \in N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}\right)$$

where $\rho = \mathrm{cov}(X_1, X_2) > 0$.

Prove that there exists a standard normal random variable $Z \in N(0,1)$ such that

$$X_1 = \rho X_2 + \sqrt{1-\rho^2} Z.$$