Stat 351 Fall 2009
Assignment #5

This assignment is due at the beginning of class on Wednesday, November 18, 2009.

1. Suppose that X_1 and X_2 are independent $N(0,1)$ random variables. Set $Y_1 = X_1 + 3X_2 - 2$ and $Y_2 = X_1 - 2X_2 + 1$.

(a) Determine the distributions of Y_1 and Y_2.

(b) Determine the distribution of $\mathbf{Y} = (Y_1, Y_2)'$.

2. Let $\mathbf{X}$ have a three-dimensional normal distribution with mean vector $\boldsymbol{\mu}$ and covariance matrix Λ given by

$$\boldsymbol{\mu} = \begin{pmatrix} 3 \\ 2 \\ -3 \end{pmatrix} \quad \text{and} \quad \Lambda = \begin{pmatrix} 2 & 1 & 3 \\ 1 & 4 & -2 \\ 3 & -2 & 8 \end{pmatrix},$$

respectively. If $Y_1 = X_1 - X_3$ and $Y_2 = 3X_2$, determine the distribution of $\mathbf{Y} = (Y_1, Y_2)'$.

3. Suppose that Y_1, Y_2, and Y_3 are independent $N(0,1)$ random variables. Set

$$\begin{aligned} X_1 &= Y_1 + Y_3, \\ X_2 &= 2Y_1 - Y_2 + 2Y_3, \\ X_3 &= 2Y_1 - 3Y_3. \end{aligned}$$

Determine the distribution of $\mathbf{X} = (X_1, X_2, X_3)'$.

4. Suppose that X_1, X_2, and X_3 are independent $N(0,1)$ random variables. Set

$$\begin{aligned} Y_1 &= X_2 - X_3, \\ Y_2 &= X_1 + 2X_3, \\ Y_3 &= X_1 - 2X_2. \end{aligned}$$

Determine the distribution of $\mathbf{Y} = (Y_1, Y_2, Y_3)'$.

5. Suppose that

$$\mathbf{X} = (X, Y)' \in N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix} \right).$$

Show that the correlation of X^2 and Y^2 is ρ^2.

Some Hints: (i) If $Z \in N(0,1)$, use the moment generating function to calculate $E(Z^4)$.

(ii) In order to calculate the higher moment involving X and Y, using conditional expectations will greatly simplify the calculation. Determine the distribution of $Y|X$. (Use Equation (6.2) in Chapter 5.) Then use Theorem 2.2 in Chapter 3.