Stat 351 Fall 2008
Assignment #2

This assignment is due at the beginning of class on Monday, September 15, 2008. You must submit all problems that are marked with an asterix (*). Let me *stress* again the *importance* of becoming comfortable with manipulating multivariate functions. It is, therefore, extremely important that at the beginning of this course you make a concerted effort to stay up-to-date. This involves reading and re-reading your class notes; reading and re-reading the relevant sections in the textbook; and taking out a pen and paper to work and re-work the calculations in your notes and the text.

In order to facilitate your learning of this material, I will continue to assign little exercises for you to do. For the most part these will be "due" by the next class. Of course, what you actually decide to do with them is your business.

1. As assigned last week, read Sections 1 through 6 of the Introduction on pages 1–10. Read Sections 7 through 10 of the Introduction on pages 10–16. We *will* be discussing sums of random variables and convolutions in great detail later in the course so feel free to skim over that part of Section 7.

2. * At the bottom of page 5, Gut asks you to "please check" the following fact. Suppose that $(\Omega, \mathcal{F}, P)$ is a probability space and that $B \in \mathcal{F}$ with $P(B) > 0$. Define a new probability Q by setting

$$Q(A) := P(A|B) = \frac{P(A \cap B)}{P(B)}.$$

Show that Q is, in fact, a probability by showing that

- $Q(\emptyset) = 0$, $Q(\Omega) = 1$,
- $Q(A^c) = 1 - Q(A)$,
- $Q(A_1 \cup A_2 \cup \cdots) = Q(A_1) + Q(A_2) + \cdots$ provided that $A_1, A_2, \ldots$ are disjoint.

3. * Fill in the details of Exercise 1.2 and show that X and Y are NOT independent, even though they are uncorrelated.

4. * Do Exercise 1.3 which is similar in spirit to Example 1.1. However, the limits of integration are much easier to handle.

5. Try Exercise 1.1. Finding the marginal distribution of (X, Y) involves a computation similar to Example 1.1. However, finding the marginal distribution of just X is much more frustrating.

6. * Suppose that X_1, X_2, and X_3 are independent and identically distributed continuous random variables with common density function $f(x)$.

(a) Compute $P\{X_1 > X_2\}$.

(b) Compute $P\{X_1 > X_2 | X_1 > X_3\}$.

(c) Compute $P\{X_1 > X_2 | X_1 < X_3\}$.

Hint: You can answer this problem easily using symmetry.