Stat 351 Fall 2008
Assignment #1

This assignment is due at the beginning of class on Monday, September 8, 2008. You must submit all problems that are marked with an asterix (*).

1. * Send me an email to say "Hello." If I have never taught you before, tell me a bit about your background in math and statistics. (I ask for everyone to send me an email so that I can create a mailing list for this class.)

2. * Let X and Y be independent random variables with $X \in \text{Unif}[1,3]$ and $Y \in \mathcal{N}(0,1)$.

(a) Determine $F_{X,Y}(x,y)$, the joint distribution function of $(X,Y)'$.

(b) Show directly (by computing the indicated partial derivative) that

$$\frac{\partial^2}{\partial x \partial y} F_{X,Y}(x,y) = f_X(x) f_Y(y).$$

Is this surprising? Why or why not?

(c) If $Z \in \text{Exp}(4)$ is independent of X and Y, determine the joint density of $(X,Y,Z)'$.

3. * On the top of page 10, Gut writes: "The joint distribution function can be expressed in terms of the joint probability function and the joint density function, respectively, in the obvious way." Write down these two (obvious) expressions for the joint distribution function.

4. * *Here is yet another STAT 251 example to show that uncorrelated random variables need not be independent.*

Suppose that $X \in \mathcal{N}(0,1)$. Let Y be independent of X with $P\{Y = 1\} = P\{Y = -1\} = 1/2$. Define the random variable Z be setting $Z = XY$.

(a) Compute $\text{cov}(X,Z)$.

(b) Show that $P\{Z \geq 1\} = P\{X \geq 1\}$. Use this fact to conclude that Z and X are NOT independent.

(c) Generalize part (b) to show that $P\{Z \geq x\} = P\{X \geq x\}$ for every $x \in \mathbb{R}$. This implies that $Z \in \mathcal{N}(0,1)$.