Stat 351 Fall 2007
Assignment #9

This assignment is due at the beginning of class on Friday, November 30, 2007. You must submit solutions to all problems. As indicated on the course outline, solutions will be graded for both content and clarity of exposition. The solutions that you submit must be neat and orderly. Do not crowd your work or write in multiple columns. Your assignment must be stapled and problem numbers clearly labelled.

1. Suppose that the random vector $\mathbf{X} = (X_1, X_2)'$ has the multivariate normal distribution

$$\mathbf{X} \in N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}\right)$$

where $\rho = \text{cov}(X_1, X_2) > 0$.

(a) Prove that there exists a standard normal random variable $Z \in N(0,1)$ such that

$$X_1 = \rho X_2 + \sqrt{1-\rho^2} Z.$$

(b) Prove that Z is independent of X_2.

2.

- Exercise 4.2, page 127
- Exercise 5.3, page 129
- Problem #27, page 147

3.

(a) If $X \in U(0,1)$, show that $-\log X$ has an exponential distribution. (What is the parameter of this exponential distribution?)

(b) Determine the density function of $\prod_{i=1}^{n} X_i$ where $X_1, \ldots, X_n$ are iid $U(0,1)$ random variables.