Stat 351 Fall 2006
Assignment #1 Solutions

2. **(a)** If $X \sim \text{Exp}(2)$, then $F_X(x) = 1 - e^{-x/2}$ for $x > 0$, and if $Y \sim \text{Unif}[0,4]$, then $F_Y(y) = \frac{y}{4}$ for $0 \le y \le 4$. Since X and Y are independent, we conclude that

$$F_{X,Y}(x,y) = F_X(x) \cdot F_Y(y) = \frac{y}{4}\left(1 - e^{-x/2}\right)$$

for $x > 0$ and $0 \le y \le 4$.

(b) We find

$$\frac{\partial^2}{\partial x \partial y} F_{X,Y}(x,y) = \frac{\partial^2}{\partial x \partial y}\left[\frac{y}{4}\left(1 - e^{-x/2}\right)\right] = \frac{1}{4} \cdot \frac{1}{2} e^{-x/2}.$$

Since $f_X(x) = \frac{1}{2}e^{-x/2}$ and $f_Y(y) = \frac{1}{4}$, we see that

$$\frac{\partial^2}{\partial x \partial y} F_{X,Y}(x,y) = f_X(x) \cdot f_Y(y)$$

as required.

(c) If $Z \sim \mathcal{N}(0,1)$ is independent of X and Y, then the joint density of (X,Y,Z) is given by

$$f_{X,Y,Z}(z,y,z) = f_X(x) \cdot f_Y(y) \cdot f_Z(z) = \frac{1}{4} \cdot \frac{1}{2} e^{-x/2} \cdot \frac{1}{\sqrt{2\pi}}\, e^{-z^2/2} = \frac{1}{\sqrt{128\pi}}\, e^{-\frac{1}{2}(x+z^2)}$$

for $x > 0$, $0 \le y \le 4$, and $-\infty < z < \infty$.

3. If X and Y are both discrete random variables, and their joint mass function is $p_{X,Y}(x,y)$, then

$$F_{X,Y}(x,y) = \sum_{x' \le x} \sum_{y' \le y} p_{X,Y}(x',y').$$

If X and Y are both continuous random variables, and their joint density function is $f_{X,Y}(x,y)$, then

$$F_{X,Y}(x,y) = \int_{-\infty}^{x} \int_{-\infty}^{y} f_{X,Y}(u,v)\, dv\, du.$$