Stat 351 Fall 2006
Assignment #8

1. Problems #9, #10, #11, #13, #14 on pages 144–145.

2. Suppose that $X \in U(0,1)$, and that the distribution of Y conditioned on $X = x$ is $N(x, x^2)$; that is, $Y|X = x \in N(x, x^2)$.

(a) Find $E(Y)$, $\text{var}(Y)$, and $\text{cov}(X, Y)$. *Hint: Conditional expectations will simplify calculations.*

(b) Prove that Y/X and X are independent.

(c) Use your results of **(a)** and **(b)** to show that

$$E\left(\frac{Y}{X}\right) = \frac{E(Y)}{E(X)}.$$

Note that in general $E(Y/X) \neq E(Y)/E(X)$.

3. Suppose that

$$\mathbf{X} = (X, Y)' \in N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}\right).$$

Show that the correlation of X^2 and Y^2 is ρ^2.

Some Hints: (i) If $Z \in N(0,1)$, use the moment generating function to calculate $E(Z^4)$. (This is a Stat 251 exercise.)

(ii) In order to calculate the higher moment involving X and Y, using conditional expectations will greatly simplify the calculation. Determine the distribution of $Y|X$. (Use Equation (6.2) on page 130.) Then use Theorem III.2.2 on page 37.

4. Suppose that the random vector $\mathbf{X} = (X_1, X_2)'$ has the multivariate normal distribution

$$\mathbf{X} \in N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}\right)$$

where $\rho = \text{cov}(X_1, X_2) > 0$. On Midterm 2, you proved that there exists a standard normal random variable $Z \in N(0,1)$ such that

$$X_1 = \rho X_2 + \sqrt{1 - \rho^2} Z.$$

Prove that Z is independent of X_2.

5.

(a) If $X \in U(0,1)$, show that $-\log X$ has an exponential distribution. (What is the parameter of this exponential distribution?)

(b) Determine the density function of $\displaystyle\prod_{i=1}^{n} X_i$ where $X_1, \ldots, X_n$ are iid $U(0,1)$ random variables.