

Math 300 Fall 2019
Assignment #1

This assignment is due at the beginning of class on Monday, September 16, 2019. Your solutions will be graded based on both correctness *and* exposition. In particular, neatness and grammar count. You must write out solutions using full sentences (including capital letters to start sentences and periods to end them) and no abbreviations. Using mathematical abbreviations such as \forall or \exists or \therefore or \Rightarrow is acceptable provided the resulting sentences are still grammatically correct. As it is unlikely that you have ever been required to pay attention to your written mathematics, this assignment will be brief.

1. Suppose that A and B are finite sets. Prove (i) $A \cup B$ is finite, and (ii) $|A \cup B| \leq |A| + |B|$.
2. Prove that if the set $E \subset \mathbb{N}$ is finite, then $E^c = \mathbb{N} \setminus E$ is countably infinite. Hint: Use Exercise 1(i) and derive a contradiction.
3. Prove that the union of a finite number of finite sets is finite. Hint: Use Exercise 1(i) combined with a proof by induction on the number of finite sets.
4. (i) Give an example of an infinite union of finite sets that is finite. (ii) Give an example of an infinite union of finite sets that is infinite.
5. Let A be a finite set. Prove that the power set of A is finite by showing that $|\mathcal{P}(A)| = 2^{|A|}$.
6. Consider the set $S = \{1, -4, 9, -16, 25, -36, 49, \dots\}$. Prove that $|S| = \aleph_0$ by explicitly finding a bijection $f : S \rightarrow \mathbb{N}$.