Stat 296 Fall 2007 Hypothesis Tests for a Difference in Means (Review Examples)

**Example 1.** Suppose that the *Saskatchewan Department of Agriculture* wants to develop plants that have an increased resistance to a particular disease. Two treatments are proposed and a random sample of 6 plants produces the following data. (Higher numbers correspond to greater resistance.)

Suppose that  $\mu_1$  denotes the true mean of plants receiving treatment #1 and that  $\mu_2$  denotes the true mean of plants receiving treatment #2. Consider testing the hypotheses  $H_0: \mu_1 = \mu_2$  vs.  $H_A: \mu_1 > \mu_2$  at the  $\alpha = 0.15$  significance level.

- (a) Based on the two-sample t test, is there significant evidence to reject  $H_0$ ?
- (b) Based on the permutation test, is there significant evidence to reject  $H_0$ ?
- (c) Based on the Wilcoxon rank-sum test, is there significant evidence to reject  $H_0$ ?

Suppose that further studies produce data as follows.

Treatment #1:	8	11	12	14	19
Treatment $#2:$	4	6	7	9	13

(d) Based on the Mann-Whitney test, is there significant evidence to reject  $H_0$ ?

**Example 2.** When designing a hypothesis test, one of the first decisions that needs to be made is which hypothesis to assign to  $H_0$  and which hypothesis to assign to  $H_A$ . One criterion that we discussed in class was to assign the hypothesis to  $H_0$  for which the consequences of a type I error are worse than the consequences of a type II error.

Consider the following hypothetical scenario. Suppose that Michael is convicted of a crime. If the judge determines that Michael is guilty of the crime, then Michael will be sent to prison. In this scenario, there are two possible hypothesis tests that can be considered.

- Hypothesis Test A:  $H_0$ : Michael is guilty vs.  $H_A$ : Michael is not guilty.
- Hypothesis Test B:  $H_0$ : Michael is not guilty vs.  $H_A$ : Michael is guilty.

In your opinion, for which of these two hypothesis tests (Test A or Test B) are the consequences of a type I error worse than the consequences of a type II error? Be sure to justify your decision, and to explain the practical significance (in terms of Michael being sent to prison) of the hypothesis test you have chosen.

**Example 3.** Suppose that we are interested in testing whether or not there is a significant difference between the exam scores for two different classes. The first class had 40 students, and the second class had 46 students. Which test would you recommend implementing? That is, which test (two sample t, permutation, Wilcoxon, Mann-Whitney) would produce the most valid results?

Group 2 (46): 46 58 47 32 50 42 41 21 48 34 56 62 25 50 55 74 72 47 47 36 44 40 42 44 48 48 34 54 41 44 22 20 73 45 46 32 42 29 77 36 29 62 39 55 50 38 72