CS 261 Fall 2011
Assignment #2

This assignment is due at the beginning of class on Monday, September 26, 2011.

On your last assignment you considered the integral equation

$$\frac{\pi}{4} = \int_0^1 \sqrt{1-x^2}\,\mathrm{d}x. \tag{$*$}$$

Using Riemann sum approximations to the integral you found approximations to π. These approximations, however, were quite poor with only $n = 6$ subintervals. Therefore, the purpose of this assignment is to use MATLAB (or OCTAVE) to compute Riemann sum approximations with much larger values of n. Recall that the left-hand Riemann sum (LHS) approximation to $(*)$ with n partition points is

$$\frac{1}{n}\sum_{j=0}^{n-1} f(\tfrac{j}{n}) = \frac{1}{n}\sum_{j=0}^{n-1}\sqrt{1-\frac{j^2}{n^2}} = \frac{1}{n^2}\sum_{j=0}^{n-1}\sqrt{n^2-j^2},$$

and the right-hand Riemann sum (RHS) approximation to $(*)$ with n partition points is

$$\frac{1}{n}\sum_{j=1}^{n} f(\tfrac{j}{n}) = \frac{1}{n}\sum_{j=1}^{n}\sqrt{1-\frac{j^2}{n^2}} = \frac{1}{n^2}\sum_{j=1}^{n}\sqrt{n^2-j^2}.$$

(a) Write a MATLAB (or OCTAVE) program to approximate the value of π by computing the values of both the LHS and the RHS with both $n = 100$ and $n = 1000$.

(b) Write a MATLAB (or OCTAVE) program to approximate the value of π by computing the values of both the LHS and the RHS with both $n = 10\ 000$ and $n = 100\ 000$.

Note that in answering questions **(a)** and **(b)** you can write either a single program to compute all four approximations simultaneously, or you can write up to four separate programs (one for each computation). The point of these problems is not for you to be writing optimal program code, but for you to write functional code! Be sure to hand in a copy of your code and indicate your final answers.

(c) The midpoint Riemann sum (MPS) approximation to $(*)$ with n partition points is

$$\frac{1}{n}\sum_{j=0}^{n-1} f(\tfrac{j}{n}+\tfrac{1}{2n}) = \frac{1}{n}\sum_{j=0}^{n-1} f(\tfrac{2j+1}{2n}) = \frac{1}{n}\sum_{j=0}^{n-1}\sqrt{1-\frac{(2j+1)^2}{(2n)^2}} = \frac{1}{2n^2}\sum_{j=0}^{n-1}\sqrt{4n^2-(2j+1)^2}.$$

Write a MATLAB (or OCTAVE) program to approximate the value of π by computing the value of the MPS with $n = 100$, $n = 1000$, $n = 10\ 000$, and $n = 100\ 000$. Be sure to hand in a copy of your code and indicate your final answers.