CS 261 Fall 2011
Assignment #1

This assignment is due at the beginning of class on Friday, September 16, 2011.

It has been known since at least 1900 BCE that the ratio of the circumference to the diameter is the same for any circle. Although the name π has only been used since 1706, the ancient Greeks (and perhaps the ancient Egyptians) were aware that the notion of π was important for architecture. As noted on Wikipedia, Archimedes (287–212 BCE) was the first to estimate π rigorously. He realized that its magnitude can be bounded from below and above by inscribing circles in regular polygons and calculating the outer and inner polygons' respective perimeters. By using the equivalent of 96-sided polygons, he proved that

$$3\tfrac{10}{71} < \pi < 3\tfrac{10}{70}.$$

The average of these values is about 3.14185.

Using Riemann sums we can give a crude approximation to π. Here is how. The function $\sqrt{1-x^2}$ for $0 \le x \le 1$ describes that part of the unit circle in the first quadrant. Since the area of a circle of radius 1 is $\pi(1)^2 = \pi$, we conclude that

$$\frac{\pi}{4} = \int_0^1 \sqrt{1-x^2}\,\mathrm{d}x. \qquad (*)$$

(a) Write down a formula to represent the left-hand Riemann sum (LHS) approximation to $(*)$ with n partition points. Simplify your formula as much as possible.

(b) Write down a formula to represent the right-hand Riemann sum (RHS) approximation to $(*)$ with n partition points. Simplify your formula as much as possible.

(c) Compute (by hand) the values of both the LHS and the RHS with both $n = 3$ and $n = 6$. Use these to derive approximations to π.

(d) Write down a formula to represent the midpoint Riemann sum (MPS) approximation to $(*)$ with n partition points. Simplify your formula as much as possible.

(e) Compute (by hand) the value of the MPS with both $n = 3$ and $n = 6$. Use these to derive approximations to π.

(f) Compute the Taylor series about the point $a = 0$ for

$$\sqrt{1+x}.$$

Use this to determine the Taylor series about the point $a = 0$ for

$$\sqrt{1-x^2}.$$

(That is, substitute $-x^2$ in for x in your Taylor series for $\sqrt{1+x}$. Be sure to simplify whenever possible.)

(g) Compute the value of

$$\int_0^1 \sqrt{1-x^2}\,\mathrm{d}x$$

using the first 5 nonzero terms in the Taylor series about the point $a = 0$ for $\sqrt{1-x^2}$. Express your answer first as a fraction then as a decimal.