Math 257 Fall 2005
Solutions to Quiz #3

(a) Briefly, ratio estimation might be appropriate because the social worker is interested in a ratio of population characteristics, and she has complete knowledge about the total number of dwellings. Furthermore, there is likely to be a direct proportion between the number of people per dwelling and the number of rooms per dwelling. If there are 0 dwellings, then there are 0 people so that the linear regression line will pass through the origin. The small sample size might be some concern, but fortunately the sampling fraction $f = 1/11 < 0.25$.

(b) We begin by finding r, the estimator of the population ratio R. Recall that $r = \bar{y}/\bar{x}$ so that

$$r = \frac{\bar{y}}{\bar{x}} = \frac{2.6}{9.2} \approx 0.283$$

From the formula for $s^2(r)$, and a little algebra, we find that

$$\begin{aligned} s^2(r) &= \frac{(1-f)}{n(n-1)\bar{x}^2} \left(\sum_{i=1}^{n} y_i^2 - 2r \sum_{i=1}^{n} y_i x_i + r^2 \sum_{i=1}^{n} x_i^2 \right) \\ &\approx \left(\frac{1 - 25/275}{25 \cdot 24 \cdot 9.2^2} \right) \left(169.0 - 2 \cdot 0.283 \cdot 522 + (0.283)^2 \cdot 2240 \right) \\ &\approx 0.000948 \end{aligned}$$

Thus, an approximate 95% CI for R is given by

$$r \pm 2s(r) \quad \text{or} \quad 0.283 \pm 0.0616.$$