Stat 257.01 Fall 2004
Assignment #10 (Selected) Solutions

(11.2) In this case, we see that the technique of interpenetrating subsamples was used. Let the index i denote the ith interviewer, $i = 1, \ldots, 10$. We easily calculate that

$$\begin{array}{ll} \overline{y}_1 = 4.8 & \overline{y}_2 = 4.8 \\ \overline{y}_3 = 7.6 & \overline{y}_4 = 5.2 \\ \overline{y}_5 = 6.2 & \overline{y}_6 = 4.6 \\ \overline{y}_7 = 4.0 & \overline{y}_8 = 5.0 \\ \overline{y}_9 = 3.6 & \overline{y}_{10} = 6.8 \end{array}$$

Thus, an estimate for the mean is given by

$$\overline{y} = \frac{1}{k} \sum \overline{y}_i = \frac{52.6}{10} = 5.26.$$

The estimated variance is given by

$$\hat{V}(\overline{y}) = \frac{N-n}{N} \cdot \frac{s_k^2}{k}$$

where

$$s_k^2 = \frac{1}{k-1} \sum (\overline{y}_i - \overline{y})^2 = \frac{1}{k-1} (\sum \overline{y}_i^2 - k\overline{y}^2) = \frac{1}{9}(290.68 - 10 \cdot 5.26^2) \approx 1.556.$$

Since N is unknown, we take the fpc to be 1 so that

$$\hat{V}(\overline{y}) \approx \frac{s_k^2}{k} \approx \frac{1.556}{10} = 0.1556.$$

A bound on the error of estimation is given by

$$B = 2\sqrt{\hat{V}(\hat{p})} \approx 0.789.$$

In other words, 5.26 ± 0.79 is an approximate 95% confidence interval for μ.

(11.9) Since the investigator used the random response technique, we find that an estimator for p, the true proportion of city dog owners who have had their dogs vaccinated against rabies, is given by $\hat{p}$ where $\hat{p}$ satisfies the equation

$$\frac{n_1}{n} = \hat{p}\theta + (1-\hat{p})(1-\theta) = \hat{p}(2\theta - 1) + (1-\theta),$$

and n_1 is the number sampled who answered "yes" to the card-question match. From the problem, we find that $n = 200$, $n_1 = 145$, $\theta = 0.8$ so that

$$\hat{p} = \frac{n_1/n}{2\theta - 1} - \frac{1-\theta}{2\theta - 1} = \frac{145/200}{0.6} - \frac{0.2}{0.6} = \frac{7}{8} = 0.875.$$

Furthermore, the estimated variance is given by

$$\hat{V}(\hat{p}) = \frac{1}{(2\theta-1)^2} \cdot \frac{1}{n} \cdot \frac{n_1}{n} \cdot \left(1 - \frac{n_1}{n}\right) = \frac{1}{(0.6)^2} \cdot \frac{1}{200} \cdot \frac{145}{200} \cdot \frac{55}{200} \approx 0.00277$$

so that a bound on the error of estimation is given by

$$B = 2\sqrt{\hat{V}(\hat{p})} \approx 0.105.$$

In other words, 0.875 ± 0.105 is an approximate 95% confidence interval for p.