Statistics 252 Midterm #2 – March 16, 2016

This exam has 2 problems and is worth a total of 50 points.

You have 50 minutes to complete this exam. Please read all instructions carefully, and check your answers. Show all work neatly and in order, and clearly indicate your final answers. Answers must be justified whenever possible in order to earn full credit. ***Unless otherwise specified, no credit will be given for unsupported answers, even if your final answer is correct.*** *Points will be deducted for incoherent, incorrect, and/or irrelevant statements. For questions with multiple parts, all parts are equally weighted.*

This exam is closed-book, although one $8\frac{1}{2} \times 11$ *double-sided page of handwritten notes is allowed. No other aids are allowed.*

You must answer all of the questions in the exam booklet provided.

Fact. *If* $Z \sim \mathcal{N}(0,1)$, *then* $P(Z > 1.645) = 0.05$.

1. (*25 points*) Suppose that $Y_1, Y_2, \ldots, Y_n$ is a random sample from a population having density

$$f(y|\theta) = \frac{\sqrt{2\theta}}{y^2\sqrt{\pi}} \exp\left\{-\frac{\theta}{2y^2}\right\}, \quad y > 0,$$

where $\theta > 0$ is a parameter.

(a) Compute the likelihood function $L(\theta)$ for this random sample.

(b) Use the factorization theorem to verify that $\displaystyle\sum_{i=1}^{n} \frac{1}{Y_i^2}$ is a sufficient statistic for the estimation of θ.

(c) Carefully verify that the maximum likelihood estimator of θ is $\hat{\theta}_{\text{MLE}} = \dfrac{n}{\displaystyle\sum_{i=1}^{n} \frac{1}{Y_i^2}}$.

(d) Find the Fisher information $I(\theta)$ in a single observation from this density.

(e) Using the standard normal approximation for the distribution of a maximum likelihood estimator based on the Fisher information, construct an approximate 90% confidence interval for θ.

2. (*25 points*) Suppose that Y_1, Y_2, Y_3 is a random sample from a population whose density is

$$f(y|\theta) = 2\theta^{-2}y, \quad 0 \le y \le \theta,$$

where $\theta > 0$ is a parameter.

(a) Determine $\hat{\theta}_{\text{MOM}}$, the method of moments estimator of θ.

(b) Carefully verify that the maximum likelihood estimator of θ is $\hat{\theta}_{\text{MLE}} = \max\{Y_1, Y_2, Y_3\}$.

(c) Determine $F_{\hat{\theta}_{\text{MLE}}}(x)$, the distribution function of $\hat{\theta}_{\text{MLE}}$, and then determine $f_{\hat{\theta}_{\text{MLE}}}(x)$, the density function of $\hat{\theta}_{\text{MLE}}$.

Consider testing $H_0 : \theta = 1$ against $H_A : \theta > 1$ by rejecting H_0 when $\hat{\theta}_{\text{MLE}} > c$ where $c > 0$ is a constant.

(d) Find c so that this test has significance level α (where $0 < \alpha < 1$).

(e) What is the power of this test (as a function of θ)?