Statistics 252 Midterm #1 – January 25, 2016

This exam has 3 problems and is worth a total of 50 points.

You have 50 minutes to complete this exam. Please read all instructions carefully, and check your answers. Show all work neatly and in order, and clearly indicate your final answers. Answers must be justified whenever possible in order to earn full credit. ***Unless otherwise specified, no credit will be given for unsupported answers, even if your final answer is correct.*** *Points will be deducted for incoherent, incorrect, and/or irrelevant statements. For questions with multiple parts, all parts are equally weighted.*

This exam is closed-book, although one $8\frac{1}{2} \times 11$ *double-sided page of handwritten notes is allowed. No other aids are allowed.*

You must answer all of the questions in the exam booklet provided.

1. (*14 points*) Suppose that $Y_1, \ldots, Y_n$ is a random sample from a population whose density is

$$f(y|\theta) = 1 - \theta\left(y - \frac{1}{2}\right), \quad 0 < y < 1,$$

where $0 < \theta < 1$ is a parameter. Let

$$\hat{\theta} := 1 - \frac{3}{n}\sum_{i=1}^{n} Y_i^2$$

be an estimator of θ.

(a) Calculate $B(\hat{\theta})$, the bias of $\hat{\theta}$.

(b) Determine the value of the constant c for which $\hat{\theta}_1 := c\,\hat{\theta}$ is an unbiased estimator of θ.

2. (*24 points*) Suppose that $Y_1, \ldots, Y_n$ is a random sample from a population whose density is

$$f(y|\theta) = 2\theta^2 y^{-3}, \quad y \geq \theta,$$

for some parameter $\theta > 1$. Let $\hat{\theta} := \min\{Y_1, \ldots, Y_n\}$.

(a) Calculate $B(\hat{\theta})$, the bias of $\hat{\theta}$.

(b) Calculate $\mathrm{MSE}(\hat{\theta})$, the mean-square error of $\hat{\theta}$.

(c) Calculate $\sigma_{\hat{\theta}}$, the standard error of $\hat{\theta}$.

3. (*12 points*) Suppose that Y_1, Y_2, Y_3 is a random sample from a population whose density is

$$f(y|\theta) = 2\theta^{-2} y, \quad 0 \leq y \leq \theta,$$

for some parameter $\theta > 0$. Let

$$\hat{\theta}_1 := \frac{3}{2} \cdot \overline{Y}$$

and let

$$\hat{\theta}_2 := \frac{7}{6} \cdot \max\{Y_1, Y_2, Y_3\}.$$

It is known that both $\hat{\theta}_1$ and $\hat{\theta}_2$ are unbiased estimators of θ. It is also known that

$$\mathrm{MSE}(\hat{\theta}_2) = \frac{\theta^2}{48}.$$

Compute $\mathrm{Eff}(\hat{\theta}_1, \hat{\theta}_2)$, the efficiency of $\hat{\theta}_2$ relative to $\hat{\theta}_1$, and determine which of these two unbiased estimators is preferable for the estimation of θ? Justify your answer.