Stat 252 Winter 2016
Solutions to Assignment #1

1. **(a)** Let X denote the size of a randomly sampled adult male's foot so that X is normally distributed with mean 25 and standard deviation 3. Therefore,

$$P(22 < X < 28) = P\left(\frac{22-25}{3} < \frac{X-25}{3} < \frac{28-25}{3}\right) = P(-1 < Z < 1) \doteq 0.683$$

where $Z \sim \mathcal{N}(0,1)$ and the last equality follows from a table of z-values.

1. **(b)** If $\overline{X}$ denotes the average size of a randomly selected adult male's foot, then X is normally distributed with mean 25 and standard deviation $3/\sqrt{100} = 0.3$. Therefore,

$$P(24.7 < \overline{X} < 25.3) = P\left(\frac{24.7-25}{0.3} < \frac{\overline{X}-25}{0.3} < \frac{25.3-25}{0.3}\right) = P(-1 < Z < 1) \doteq 0.683$$

where $Z \sim \mathcal{N}(0,1)$ and the last equality follows from a table of z-values as in **(a)** above.

2. **(a)** Write the values in order: 150, 180, 190, 230, 250, 250, 280, 300, 340, 380. The sample median is just the average of the two middle numbers. Since these two numbers are both 250, the sample median is 250. The sample mean is a simple calculation: $(150 + 180 + 190 + 230 + 250 + 250 + 280 + 300 + 340 + 380)/10 = 255$. The sample standard deviation is calculated just as easily:

$$\sqrt{\frac{697300 - \frac{2550^2}{10}}{9}} \doteq 72.$$

2. **(b)** For Bright Idea Lighting, if X_1 denotes the lifetime of a randomly selected bulb, then X_1 is normal with mean 262 and standard deviation 41 implying

$$P(X_1 > 350) = P\left(Z > \frac{350-262}{41}\right) \doteq P(Z > 2.15) \doteq 0.0158.$$

For The Electric Company, if X_2 denotes the lifetime of a randomly selected bulb, then X_2 is approximately normal with mean 255 and standard deviation 72 implying

$$P(X_2 > 350) \doteq P\left(Z > \frac{350-255}{72}\right) \doteq P(Z > 1.32) \doteq 0.0934$$

where in both cases $Z \sim \mathcal{N}(0,1)$ and using a table of z-values.

2. **(c)** An approximate 95% confidence interval for the true mean lifetime of The Electric Company's light bulbs is given by

$$\overline{X} \pm t_{0.025,n-1}\frac{S}{\sqrt{n}} \quad \text{or} \quad 255 \pm 2.262\frac{72}{\sqrt{10}} \quad \text{or} \quad [204, 307].$$

2. **(d)** Since the mean lifetime of Bright Idea Lighting light bulbs is 262, and since 262 lies in the 95% confidence interval constructed in **(c)**, we conclude that there is *no* significant difference in mean lifetimes for these two companies' light blubs.

3. Let μ_1 denote the mean waiting time for *Cheap-O-Lube* customers last year, and let μ_2 denote the mean waiting time for *Cheap-O-Lube* customers this year. We are interested in testing the hypotheses

$$H_0 : \mu_1 - \mu_2 \leq 0 \quad \text{vs.} \quad H_1 : \mu_1 - \mu_2 > 0.$$

Since the population variances are unknown, we use a two sample t-test. Thus, our test statistic is

$$T = \frac{\overline{X}_1 - \overline{X}_2}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}} = \frac{4.5 - 3.5}{\sqrt{\frac{1^2}{200} + \frac{1^2}{180}}} \doteq 9.73.$$

Since there are df $= n_1 + n_2 - 2 = 378$ degrees of freedom, we can approximate this t-test by a z-test. From a z-table, the critical value corresponding to $\alpha = 0.05$ is 1.645. Since $9.73 > 1.645$ we reject H_0 and conclude that there is overwhelming evidence to suggest that *Cheap-O-Lube* customers are waiting less this year.