Stat 252 Winter 2016
Assignment #5

This assignment is due at the beginning of class on Friday, March 4, 2016. Your solutions will be graded based on both correctness *and* exposition. In particular, neatness and grammar count. You must write out solutions using full sentences (including capital letters to start sentences and periods to end them) and no abbreviations. That is, symbols such as $\therefore$ and $\Rightarrow$ are forbidden; write out the full words *therefore* and *implies* in their place.

1. Consider a random variable Y with density function

$$f(y|\theta) = \theta^2 e^{-\theta^2 y}, \quad y > 0,$$

where $\theta > 0$ is a parameter. Assume that $0 < \alpha_1 < \frac{1}{2}$ and $0 < \alpha_2 < \frac{1}{2}$. Using the pivotal quantity $U = \theta^2 Y$, verify that

$$\left[\sqrt{\frac{-\log(1-\alpha_1)}{Y}}, \sqrt{\frac{-\log(\alpha_2)}{Y}} \right]$$

is a confidence interval for θ with coverage probability $1 - (\alpha_1 + \alpha_2)$.

2. Consider a random variable Y with density function

$$f(y|\theta) = 2\theta^{-2} y, \quad 0 \le y \le \theta$$

where $\theta > 0$ is a parameter. Use the pivotal method to verify that if $0 < \alpha < 1$, then

$$\left[\frac{Y}{\sqrt{1-\alpha/2}}, \frac{Y}{\sqrt{\alpha/2}} \right]$$

is a confidence interval for θ with coverage probability $1 - \alpha$.

3. Consider a random variable Y with density function

$$f(y|\theta) = \frac{e^{(y-\theta)}}{\left[1 + e^{(y-\theta)}\right]^2}, \quad -\infty < y < \infty,$$

where $-\infty < \theta < \infty$ is a parameter. Use the pivotal method to verify that if $0 < \alpha_1 < 1/2$ and $0 < \alpha_2 < 1/2$, then

$$\left[Y - \log\left(\frac{1-\alpha_2}{\alpha_2}\right), Y - \log\left(\frac{\alpha_1}{1-\alpha_1}\right) \right]$$

is a confidence interval for θ with coverage probability $1 - (\alpha_1 + \alpha_2)$.

4. Suppose that $Y_1, \ldots, Y_n$ is a random sample from a Uniform$[0, \theta]$ population where $\theta > 0$ is a parameter. Let $\hat{\theta} = \max\{Y_1, \ldots, Y_n\}$. Use the pivotal method to construct a symmetric 90% confidence interval for θ based on $\hat{\theta}$.