Stat 252.01 Winter 2007 Assignment #10 Solutions (Corrections)

5. To find the method of moments estimators for λ and θ , we must solve the system of equations

$$\mathbb{E}(Y) = \overline{Y} \text{ and } \mathbb{E}(Y^2) = \frac{1}{n} \sum_{i=1}^n Y_i^2$$

Since $\mathbb{E}(Y) = \theta$ and $\operatorname{Var} Y = 2\lambda^{-2}$, we find $\mathbb{E}(Y^2) = \operatorname{Var} Y - [\mathbb{E}(Y)]^2 = 2\lambda^{-2} + \theta^2$. Thus,

$$\theta = \overline{Y}$$
 and $2\lambda^{-2} + \theta^2 = \frac{1}{n} \sum_{i=1}^n Y_i^2$

and so some trivial algebra gives

$$\hat{\theta}_{\text{MOM}} = \overline{Y} \text{ and } \hat{\lambda}_{\text{MOM}} = \sqrt{\frac{2n}{\sum_{i=1}^{n} Y_i^2 - n\overline{Y}^2}}.$$