Statistics 252 Midterm #3 – April 3, 2006

This exam has 4 problems on 4 numbered pages, and is worth 35 points.

You have 50 minutes to complete this exam. Please read all instructions carefully, and check your answers. Show all work neatly and in order, and clearly indicate your final answers. Answers must be justified whenever possible in order to earn full credit. ***Unless otherwise specified, no credit will be given for unsupported answers, even if your final answer is correct.*** *Points will be deducted for incoherent, incorrect, and/or irrelevant statements.*

This exam is closed-book, except that one $8\frac{1}{2}\times 11$ *double-sided page of handwritten notes is permitted. No other aids are allowed. Copies of the required tables will be provided.*

You must answer all of the questions in the space provided. Note that blank space is NOT an indication of a question's difficulty.

Name: ____

Instructor: Michael Kozdron

Problem	Score
1	
2	
3	
4	

TOTAL: _____________

1. (*8 points*) Suppose that $Y_1, \ldots, Y_5$ are independent and identically distributed $\mathcal{N}(\mu, \sigma^2)$ random variables. As usual, let

$$\overline{Y} = \frac{1}{5}\sum_{i=1}^{5} Y_i \quad \text{and} \quad S^2 = \frac{1}{4}\sum_{i=1}^{5}(Y_i - \overline{Y})^2.$$

It is known that $\mu = 0$, but σ^2 is unknown. You want to test $H_0 : \sigma = 1$ against $H_A : \sigma > 1$ by rejecting H_0 when $S^2 > 1.945$. (*You may consider your calculations accurate to two decimal places when consulting the appropriate table(s).*)

(a) Verify that this test has significance level $\alpha = 0.10$.

(b) Using the test determined in **(a)**, find the power of the test when $\sigma = 3.3$.

2. (*15 points*) Suppose that $Y_1, \ldots, Y_n$ are independent and identically distributed random variables with each Y_i having density function

$$f(y|\theta) = \frac{\theta^2}{y^3}\exp(-\theta/y), \quad y > 0$$

for some parameter $\theta > 0$.

(a) Compute the likelihood function $L(\theta)$ for this random sample.

(b) Show that the maximum likelihood estimator of θ is $\hat{\theta}_{\text{MLE}} = \dfrac{2n}{\sum_{i=1}^{n} \frac{1}{Y_i}}$.

(c) Show that $\sum_{i=1}^{n} \frac{1}{Y_i}$ is a sufficient statistic for the estimation of θ.

(continued)

(d) Explain why $\hat{\theta}_{\text{MLE}}$ must also be a sufficient statistic for the estimation of θ.

(e) Find the Fisher information $I(\theta)$ in a single observation from this density.

(f) Starting with an approximate confidence interval for θ based on the Fisher information, determine the rejection region for the test of the hypothesis $H_0 : \theta = \theta_0$ against $H_A : \theta \neq \theta_0$ at the (approximate) significance level $\alpha = 0.05$.

3. (*4 points*) Assume that the outcome of an experiment is a single random variable Y. A 90% confidence interval for a parameter θ has the form $(Y-2,\, Y+3)$. From this, determine a rejection rule for testing $H_0 : \theta = 4$ against $H_A : \theta \neq 4$ at significance level 0.10.
(Hint: Use the confidence interval–hypothesis test duality.)

4. (*8 points*) Let Y be a Uniform$(0, \theta)$ random variable. Consider testing $H_0 : \theta = 1$ against $H_A : \theta > 1$ by rejecting H_0 when $Y > c$.

(a) Find c so that this test has significance level 0.05.

(b) What is the power of the test in **(a)** (as a function of θ)?