Statistics 252 Midterm #2 – March 3, 2006

This exam has 4 problems on 4 numbered pages, and is worth 35 points.

You have 50 minutes to complete this exam. Please read all instructions carefully, and check your answers. Show all work neatly and in order, and clearly indicate your final answers. Answers must be justified whenever possible in order to earn full credit. ***Unless otherwise specified, no credit will be given for unsupported answers, even if your final answer is correct.*** *Points will be deducted for incoherent, incorrect, and/or irrelevant statements.*

This exam is closed-book, except that one $8\frac{1}{2} \times 11$ *double-sided page of handwritten notes is permitted. No other aids are allowed.*

You must answer all of the questions in the space provided. Note that blank space is NOT an indication of a question's difficulty.

Name: ____

Instructor: Michael Kozdron

Problem	Score
1	
2	
3	
4	

TOTAL: _____________

1. (*10 points*)

(a) Suppose that a random variable Y has density function

$$f(y|\theta) = (\theta + 1)y^{\theta}, \quad 0 \le y \le 1.$$

Determine the Fisher information $I(\theta)$ for this random variable.

(b) Let $Y_1, \ldots, Y_n$ be independent and identically distributed random variables each with the density $f(y|\theta)$ given in **(a)**. Determine $\hat{\theta}_{\text{MOM}}$, the method of moments estimator of θ.

2. (*7 points*) Consider a random variable Y with density function

$$f(y|\theta) = 2\theta^{-2}y, \quad 0 \leq y \leq \theta$$

for some parameter $\theta > 0$. Use the pivotal method to verify that if $0 < \alpha < 1$, then

$$\left(\frac{Y}{\sqrt{1-\alpha/2}}, \frac{Y}{\sqrt{\alpha/2}} \right)$$

is a confidence interval for θ with coverage probability $1 - \alpha$.

3. (*10 points*) Recall that a discrete random variable Y is said to be Poisson with parameter $\theta > 0$ if the density (also called probability mass function) of Y is

$$f(y|\theta) = \frac{\theta^y e^{-y}}{y!}, \quad y = 0, 1, 2, \ldots.$$

Recall further that if Y is Poisson(θ), then $E(Y) = \theta$ and $\mathrm{Var}(Y) = \theta$.

(a) It turns out that the same formula for the Fisher information can be used for discrete random variables. Show that if Y is a Poisson random variable with parameter θ, then $I(\theta) = \frac{1}{\theta}$.

For parts **(b)**, **(c)**, **(d)**, and **(e)** below, suppose that $Y_1, Y_2, \ldots, Y_n$ are independent and identically distributed Poisson(θ) random variables.

(b) Show that $\hat{\theta}_{\mathrm{MOM}}$, the method of moments estimator of θ, is $\hat{\theta}_{\mathrm{MOM}} = \overline{Y} = \frac{Y_1 + \cdots + Y_n}{n}$.

(c) Show that $\hat{\theta}_{\mathrm{MOM}}$ is an unbiased estimator of θ.

(d) Calculate $\mathrm{Var}(\hat{\theta}_{\mathrm{MOM}})$.

(e) Explain why $\hat{\theta}_{\mathrm{MOM}}$ must be the minimum variance unbiased estimator (MVUE) of θ.

4. (*8 points*)

(a) Let $Y_1, Y_2, \ldots, Y_n$ be independent and identically distributed random variables each with mean p and variance $p(1-p)$. Here p is an unknown parameter between 0 and 1. If

$$\hat{p} = \frac{Y_1 + \cdots + Y_n}{n}$$

then it is known that $\hat{p}$ is an unbiased estimator of p. Show that the maximum value of $\mathrm{Var}(\hat{p})$, the variance of $\hat{p}$, occurs when $p = 1/2$.

(b) Suppose that $\hat{\theta}_1$ and $\hat{\theta}_2$ are both unbiased estimators of θ. Explain how $\mathrm{Eff}(\hat{\theta}_1, \hat{\theta}_2)$, the relative efficiency of $\hat{\theta}_1$ to $\hat{\theta}_2$, can be used to decide which of these two estimator is preferable.