Statistics 252 Midterm #1 – January 30, 2006

This exam has 5 problems and 6 numbered pages.

You have 50 minutes to complete this exam. Please read all instructions carefully, and check your answers. Show all work neatly and in order, and clearly indicate your final answers. Answers must be justified whenever possible in order to earn full credit. ***Unless otherwise specified, no credit will be given for unsupported answers, even if your final answer is correct.*** *Points will be deducted for incoherent, incorrect, and/or irrelevant statements.*

This exam is closed-book, except that one $8\frac{1}{2}\times 11$ double-sided page of handwritten notes is permitted. No other aids are allowed. A copy of Table 4 will be provided.

You must answer all of the questions in the space provided. Note that blank space is NOT an indication of a question's difficulty.

Name: ______________________________________

Instructor: Michael Kozdron

Problem	Score
1	
2	
3	
4	
5	

TOTAL: ____________

1. (*9 points*) Suppose that Y_1, Y_2, Y_3 is a random sample where the density of each random variable Y_i is

$$f_Y(y) = 2\theta^{-2}y, \quad 0 \leq y \leq \theta$$

for some parameter $\theta > 0$.

(a) Show that $\hat{\theta}_1 = \frac{3}{2} \cdot \overline{Y}$ is an unbiased estimator of θ.

(b) Show that $\hat{\theta}_2 = \frac{7}{6} \cdot \max\{Y_1, Y_2, Y_3\}$ is an unbiased estimator of θ.

(continued)

(c) Which of the two unbiased estimators given in **(a)** and **(b)** is preferable for the estimation of θ? Justify your answer.

2. (*5 points*) For Saskatchewan residents, the only way to return empty cans for deposit is to bring them to a SARCAN depot. The time that an individual customer spends with the clerk is well-modelled by a normal distribution with mean 10 and variance 8 (measured in minutes). Suppose that 3 people arrive at a SARCAN depot at the same time, and that there is only one clerk. What is the probability that the third person will wait more than 24 minutes before being served? (In other words, what is the probability that it will take the clerk more than 24 minutes to serve the first two people?)

3. (*6 points*) Recall that a discrete random variable Y is said to be Poisson with parameter $\lambda > 0$ if the density (also called probability mass function) of Y is

$$f_Y(y) = \frac{\lambda^y e^{-y}}{y!}, \quad y = 0, 1, 2, \ldots.$$

Recall further that if Y is Poisson(λ), then $E(Y) = \lambda$ and $\text{Var}(Y) = \lambda$.

Suppose that $Y_1, Y_2, \ldots, Y_n$ are independent and identically distributed Poisson(λ) random variables.

(a) Verify that $\overline{Y} = \dfrac{Y_1 + \cdots + Y_n}{n}$ is an unbiased estimator of λ.

(b) Verify that $S^2 = \dfrac{1}{n-1}\displaystyle\sum_{i=1}^{n}(Y_i - \overline{Y})^2$ is an unbiased estimator of λ.
Hint: Expand the sum and use properties of expected value and variance.

4. (*9 points*) Suppose that Y_1, Y_2, and Y_3 are independent random variables where $E(Y_1) = 4\theta$, $\text{Var}(Y_1) = 1$, and $E(Y_2) = -\theta$, $\text{Var}(Y_2) = 4$, and $E(Y_3) = -2\theta$, $\text{Var}(Y_3) = 3$, for some unknown parameter $\theta > 0$. Consider the estimator

$$\hat{\theta} = \frac{\alpha}{2}Y_1 + (\alpha - 1)Y_2 + \frac{\alpha}{2}Y_3$$

where α is a constant between 0 and 1.

(a) Calculate the bias and variance of $\hat{\theta}$.

(i) bias:

(ii) variance:

(b) What value of α should you use to minimize the variance of $\hat{\theta}$?

5. (*6 points*) Each of the following require short written answers. Very clear and brief answers are required for full credit.

(a) In Stat 151, the term *statistic* was used to refer to a single number computed from data. In Stat 252, however, we defined *statistic* as both a single random variable *and* as a number computed from data. In the context of Stat 252, explain this dual nature of *statistic*.

(b) In the context of Stat 252, explain the difference between a *parameter* and an *estimator*, and explain how an estimator is used to predict, or estimate, a parameter.