Stat 252 Winter 2006
January 25, 2006

1. Suppose that $Y_1, Y_2, \ldots, Y_{27}$ are a random sample of exponential random variables with parameter $\theta > 0$. That is, the density of each Y_i is

$$f(y) = \frac{1}{\theta} e^{-y/\theta}, \quad y > 0.$$

(a) Show that $\hat{\theta}_1 = Y_{27}$ is an unbiased estimator of θ.

(b) Show that $\hat{\theta}_2 = 27 \cdot \min\{Y_1, \ldots, Y_{27}\}$ is an unbiased estimator of θ.

(c) Show that $\hat{\theta}_3 = \overline{Y}$ is an unbiased estimator of θ.

(d) Which of the three unbiased estimators given in **(a)**, **(b)**, and **(c)** is preferable for the estimation of θ? Justify your answer.

2. A medicinal herb growing operation maintains a generator to power 25 heat lamps in its greenhouse so that when one lamp fails, another immediately takes over. (Only one lamp is lit at a time.) The heat lamps operate independently, and each has a lifetime which is normally distributed as $\mathcal{N}(50, 4)$ (in hours). If the greenhouse is not checked for 1300 hours after the generator is turned on, what is the probability that a lamp will be burning at the end of the 1300-hour period?