Stat 252.01 Winter 2005
Assignment #4 Solutions

(10.2) (a) It might be helpful to have clear null and alternative hypotheses in mind. Therefore, let H_0 be "the drug dosage level induces sleep in 80% of people suffering from insomnia" and let H_A be "the drug dosage level induces sleep in less than 80% of people suffering from insomnia." Hence, a Type I error (reject H_0 when H_0 is true) occurs if the experimenter concluded that the drug dosage level induces sleep in less than 80% of the people suffering from insomnia when, in fact, drug dosage level does induce sleep in 80% of insomniacs.

(b) We find that the significance level α is given by

$$\alpha = P(\text{Type I error}) = P_{H_0}(\text{reject } H_0) = P(Y \leq 12 \,|\, p = 0.8) = \sum_{k=0}^{12} \binom{20}{k} 0.8^k 0.2^{20-k} \approx 0.032.$$

(c) A Type II error occurs if the experimenter concluded that the drug dosage level induces sleep in 80% of the people suffering from insomnia when, in fact, fewer than 80% of insomniacs experience relief.

(d) When $p = 0.6$, we find

$$\beta = P(\text{Type II error}) = P_{H_A}(\text{accept } H_0) = P(Y > 12 \,|\, p = 0.6) = \sum_{k=13}^{20} \binom{20}{k} 0.6^k 0.4^{20-k} \approx 0.416.$$

(e) When $p = 0.4$, we find

$$\beta = P(\text{Type II error}) = P_{H_A}(\text{accept } H_0) = P(Y > 12 \,|\, p = 0.4) = \sum_{k=13}^{20} \binom{20}{k} 0.4^k 0.6^{20-k} \approx 0.021.$$

(10.3) (a) We must find c so that $P(Y \leq c \,|\, p = 0.8) \approx 0.01$. From Table 1 in the appendix, we find that $c = 11$ suffices. (In fact, to the accuracy allowed by the table, $P(Y \leq 11 \,|\, p = 0.8) = 0.01$.)

(b) With the rejection region given as $\{Y \leq 11\}$, we find that for $p = 0.6$,

$$\beta = P(Y > 11 \,|\, p = 0.6) = \sum_{k=12}^{20} \binom{20}{k} 0.6^k 0.4^{20-k} \approx 0.596.$$

(c) With the rejection region given as $\{Y \leq 11\}$, we find that for $p = 0.4$,

$$\beta = P(Y > 11 \,|\, p = 0.4) = \sum_{k=12}^{20} \binom{20}{k} 0.4^k 0.6^{20-k} \approx 0.057.$$