Math 302.102 Midterm #2 – November 15, 2010

This exam has 4 problems on 4 numbered pages and is worth a total of 50 points.

You have 50 minutes to complete this exam. Please read all instructions carefully, and check your answers. Show all work neatly and in order, and clearly indicate your final answers. Answers must be justified whenever possible in order to earn full credit. ***Unless otherwise specified, no credit will be given for unsupported answers, even if your final answer is correct.*** *Points will be deducted for incoherent, incorrect, and/or irrelevant statements.*

This exam is closed-book, although a handheld calculator is permitted. No other aids are allowed.

You must answer all of the questions in the space provided. Note that blank space is NOT an indication of a question's difficulty.

Name: ____________________________________ Instructor: Michael Kozdron

Problem	Score
1	
2	
3	
4	

TOTAL: ____________

1. (*20 points*) Suppose that the distribution function of the continuous random variable X is

$$F_X(x) = \begin{cases} 1 - x^{-a}, & x > 1, \\ 0, & x \le 1 \end{cases}$$

where $a > 2$ is a parameter.

(a) Determine $f_X(x)$, the density function of X.

(b) Compute $\mathbb{E}(X)$

(c) Compute $\mathrm{var}(X)$.

(d) Compute $\mathbf{P}\{X \ge tx \,|\, X \ge x\}$ for any $t > 1$.

(e) Suppose that $Y = X^a$. Determine the density function of Y.

2. (*10 points*) Suppose that the density function of the continuous random variable X is

$$f_X(x) = \begin{cases} \frac{1}{2}(x+1), & -1 < x \leq 1, \\ 0, & \text{otherwise.} \end{cases}$$

(a) Compute $\mathbf{P}\left\{-2 < X \leq \frac{1}{2}\right\}$.

(b) Suppose that $Y = |X|$. Determine the density function of Y.

3. (*10 points*) Suppose that the lifetime X (in years) of a circuit board produced by the Toshabi Company has density function

$$f(x) = \frac{2}{(x+1)^3}$$

for $x \geq 0$. Suppose further that the lifetimes of distinct Toshabi circuit boards are independent. Alice requires three circuit boards for her computer system to operate. Furthermore, all three circuit boards must be working in order for her computer system to operate. Determine the expected amount of time until Alice's computer system stops operating.

4. (*10 points*) Yolanda and Xavier agree to meet at a party. Suppose that the party starts at time 0, and that we measure all times in hours. Yolanda, who wants to be fashionably late, arrives at a random time Y which has the density function

$$f_Y(y) = 1 \text{ for } 0 \leq y \leq 1.$$

Xavier, who is distracted by a televised hockey game, arrives at a random time X which has the density function

$$f_X(x) = e^{-x} \text{ for } x \geq 0.$$

Yolanda will get impatient and leave if Xavier arrives more than one hour after she does. If their arrival times X and Y are assumed to be independent, what is the probability Yolanda will leave before Xavier arrives?