Math/Stat 251 Fall 2015
Assignment #2

Solutions should be completed, but not submitted, by Wednesday, September 23, 2015.

1. Suppose that the sample space S consists of four outcomes, say $S = \{a, b, c, d\}$.

(a) Explicitly list all of the possible *events*.

(b) Assuming that each outcome is equally likely, give a succinct formula for $\mathbf{P}\{A\}$ for any event A. (In other words, do not simply list $\mathbf{P}\{A\}$ for each possible A. As you know from part **(a)**, there are a lot of events to list.)

2. Suppose that there are three identical coins, each of which shows heads with probability $3/5$ and shows tails with probability $2/5$. Call the three coins A, B, and C. Coin A counts 10 points if a head appears and 2 points if a tail appears. Coin B counts 4 points if a head appears and 4 points if a tail appears. Coin C counts 3 points if a head appears and 20 points if a tail appears. You and your opponent each choose a coin; you cannot choose the same coin. Each of you tosses your coin once and the person with the larger score wins (a desirable prize). Would you prefer to be the first or the second to choose a coin? Justify your answer probabilistically.

3. A typical slot machine in a Saskatchewan casino has three wheels, each marked with twenty symbols at equal spacings around the wheel. The machine is engineered so that on each play the three wheels spin independently, and each wheel is equally likely to show any one of its twenty symbols when it stops spinning. On the central wheel, nine out of twenty symbols are bells, while there is only one bell on the left wheel and one bell on the right wheel. The machine pays out the jackpot only if the wheels come to rest with each wheel showing a bell.

(a) Calculate the probability of hitting the jackpot.

(b) Calculate the probability of getting exactly two bells.

(c) Suppose instead that there were three bells on the left, one in the middle, and three on the right. How would this affect the probabilities in **(a)** and **(b)**? Explain why the casino might find the 1–9–1 machine more profitable than a 3–1–3 machine.

4. Suppose that two players A and B are playing the following game. A fair coin is tossed repeatedly until either the sequence TTT occurs in which case player A wins, or the sequence HTT occurs in which case player B wins. What is the probability that player B wins?

5. Consider the following circuit with four switches labelled 1, 2, 3, 4. Assume that switch i is closed with probability p_i and open with probability $q_i = 1 - p_i$. Assume further that the switches function independently. Find a formula for the probability that a current can flow from left-to-right through the circuit. (Note that in order for current to flow there must be at least one closed path from left-to-right.)