Stat 160 Fall 2008
Solutions to Assignment #6

11.32 As noted in the first box on page 278, the mean and standard deviation of $\overline{x}$ are μ and $\sigma/\sqrt{n}$, respectively. If the heights of young men have standard deviation 2.8 inches, then the sample mean of a SRS of n young men has standard deviation

$$\frac{2.8}{\sqrt{n}}.$$

If we want to reduce the standard deviation of the sample mean to 0.5 inches, then we must choose n to satisfy

$$\frac{2.8}{\sqrt{n}} = 0.5.$$

Solving for $\sqrt{n}$ gives $\sqrt{n} = 2.8/0.5 = 5.6$ and so

$$n = 5.6^2 = 31.36.$$

Since we cannot have fractional people, we must take $n = 32$. (Note that taking $n = 31$ is not suitable for the resulting standard deviation would then be larger than 0.5.)

11.34 (a) If X denotes the height of a male student on campus, then X has a normal $N(70, 2.8)$ distribution. Thus, the required probability is

$$\begin{aligned} P(69 \le X \le 71) &= P\left(\frac{69-70}{2.8} \le \frac{X-70}{2.8} \le \frac{71-70}{2.8}\right) \\ &= P(-0.36 \le Z \le 0.36) \\ &= 0.6406 - 0.3594 \\ &= 0.2812 \end{aligned}$$

using Table A.

11.34 (b) If $\overline{x}$ denotes the mean heights of $n = 32$ students, then $\overline{x}$ has a normal $N(70, 2.8/\sqrt{32}) = N(70, 0.495)$ distribution. Thus, the required probability is

$$\begin{aligned} P(69 \le \overline{x} \le 71) &= P\left(\frac{69-70}{0.495} \le \frac{\overline{x}-70}{0.495} \le \frac{71-70}{0.495}\right) \\ &= P(-2.02 \le Z \le 2.02) \\ &= 0.9783 - 0.0217 \\ &= 0.9566 \end{aligned}$$

using Table A.

Alternatively, you can answer this question without knowing the sample size $n = 32$. Since n is chosen to make the margin of error of $\overline{x}$ equal to 0.5, we have that $\overline{x}$ has a normal $N(70, 0.5)$ distribution. Thus, the required probability is

$$\begin{aligned} P(69 \le \overline{x} \le 71) &= P\left(\frac{69-70}{0.5} \le \frac{\overline{x}-70}{0.5} \le \frac{71-70}{0.5}\right) \\ &= P(-2 \le Z \le 2) \\ &= 0.9772 - 0.0228 \\ &= 0.9544 \end{aligned}$$

using Table A.

11.40 Let X denote the return on US common stock so that X has a normal $N(8.7, 20.2)$ distribution. If $\overline{x}$ denotes Andrew's average over the next $n = 40$ years, then $\overline{x}$ has a normal $N(8.7, 20.2/\sqrt{40}) = N(8.7, 3.1939)$ distribution. Thus, the probability that the mean return over the next 40 years exceeds 10% is

$$P(\overline{x} > 10) = P\left(\frac{\overline{x} - 8.7}{3.1939} > \frac{10 - 8.7}{3.1939}\right) = P(Z > 0.41) = 1 - 0.6591 = 0.3409$$

using Table A. The probability that the mean return over the next 40 years is less than 5% is

$$P(\overline{x} < 5) = P\left(\frac{\overline{x} - 8.7}{3.1939} < \frac{5 - 8.7}{3.1939}\right) = P(Z < -1.16) = 0.1230$$

also using Table A.

12.30 (a) The probability that the portfolio goes up three consecutive years is

$$P(\text{Up Up Up}) = P(\text{Up}) \cdot P(\text{Up}) \cdot P(\text{Up}) = (0.65)(0.65)(0.65) = 0.274625.$$

12.30 (b) The probability that the portfolio goes in the same direction for three consecutive years is

$$\begin{aligned} P(\text{Up Up Up or Down Down Down}) &= P(\text{Up Up Up}) + P(\text{Down Down Down}) \\ &= P(\text{Up}) \cdot P(\text{Up}) \cdot P(\text{Up}) + P(\text{Down}) \cdot P(\text{Down}) \cdot P(\text{Down}) \\ &= (0.65)^3 + (0.35)^3 \\ &= 0.3175. \end{aligned}$$

13.24 (a) The distribution of the number who have visited an online auction site in the past month is binomial with parameters $n = 12$ and $p = 0.50$.

13.24 (b) The probability that exactly 8 of the 12 have visited an auction site in the past month is

$$P(8 \text{ of } 12) = \binom{12}{8}(0.5)^8(1 - 0.5)^{12-8} = \binom{12}{8}(0.5)^8(0.5)^4 = \binom{12}{8}(0.5)^{12} = 0.1208496.$$

13.26 (a) The distribution of X is binomial with parameters $n = 5$ and $p = 0.65$.

13.26 (b) The possible values of X are 0, 1, 2, 3, 4, and 5.

13.26 (c) We find

$$P(X = 0) = \binom{5}{0}(0.65)^0(1 - 0.65)^{5-0} = (0.35)^5 = 0.005252187,$$

$$P(X = 1) = \binom{5}{1}(0.65)^1(1 - 0.65)^{5-1} = 5(0.65)(0.35)^4 = 0.04877031,$$

$$P(X = 2) = \binom{5}{2}(0.65)^2(1 - 0.65)^{5-2} = 10(0.65)^2(0.35)^3 = 0.1811469,$$

$$P(X = 3) = \binom{5}{3}(0.65)^3(1 - 0.65)^{5-3} = 10(0.65)^3(0.35)^2 = 0.3364156,$$

$$P(X = 4) = \binom{5}{4}(0.65)^4(1 - 0.65)^{5-4} = 5(0.65)^4(0.35) = 0.3123859,$$

$$P(X = 5) = \binom{5}{5}(0.65)^5(1 - 0.65)^{5-5} = (0.65)^5 = 0.1160291.$$

13.26 (d) The mean is $\mu = np = 5 \times 0.65 = 3.3$ and the standard deviation is $\sigma = \sqrt{np(1-p)} = \sqrt{5(0.65)(0.35)} = 1.066536$.

13.30 (a) Since $n = 100$ and $p = 0.75$, if we denote Jodi's score by X, then the distribution of X is approximately normal with mean $\mu = np = 100 \times 0.75 = 75$ and standard deviation $\sigma = \sqrt{np(1-p)} = \sqrt{100 \times 0.75 \times 0.25} = \sqrt{18.75} = 4.33$. Thus, using the normal approximation to the binomial, the required probability is

$$\begin{aligned} P(X \leq 70) &= P\left(\frac{X-75}{4.33} \leq \frac{70-75}{4.33}\right) \\ &= P(Z \leq -1.15) \\ &= 0.1251 \end{aligned}$$

using Table A.

13.30 (b) If the test instead contains 250 questions and X again denotes Jodi's score, then the distribution of X is approximately normal with mean $\mu = np = 250 \times 0.75 = 187.5$ and standard deviation $\sigma = \sqrt{np(1-p)} = \sqrt{250 \times 0.75 \times 0.25} = \sqrt{46.875} = 6.8465$. If Jodi is to score 70% or lower, then her score out of 250 must be less than $250 * 0.70 = 175$. Thus, using the normal approximation to the binomial, the required probability is

$$\begin{aligned} P(X \leq 175) &= P\left(\frac{X-187.5}{6.8465} \leq \frac{175-187.5}{6.8465}\right) \\ &= P(Z \leq -1.83) \\ &= 0.0336 \end{aligned}$$

using Table A.