Stat 151.003 (Kozdron) Fall 2006
Central Limit Theorem
October 25, 2006

Example. A large population of seeds of the princess bean *Phaseolus vulgaris* is to be sampled. The weights of the seeds in the population are known to have mean $\mu = 500$ mg and standard deviation $\sigma = 120$ mg. Suppose that a random sample of 100 seeds is to be weighed. Let $\overline{X}$ be the mean weight of the 100 seeds.

(a) What is the approximate distribution of $\overline{X}$?

(b) What is the probability that $\overline{X}$ will be greater than 520 mg?

(c) What is the weight w such that the probability that $\overline{X}$ is greater than w is only 0.01?

Example. Bright Idea Lighting tests their light bulbs, and finds that they have a mean life of 262 hours, with a standard deviation of 41 hours. They test a sample of light bulbs of their rival, The Electric Company, and get that they last 340, 190, 150, 280, 250, 180, 380, 300, 250, and 230 hours.

(a) Find the median, mean, and standard deviation of the life of The Electric Company's light bulbs.

(b) Which brands' light bulbs have the higher mean life?

(c) Assuming the distribution of bulb life of both companies follows a normal distribution, how likely is each company to produce a light bulb that lasts 350 hours?

Solution. **(a)** Write the values in order: 150, 180, 190, 230, 250, 250, 280, 300, 340, 380. The median is just the mean of the two middle numbers. Since these two numbers are both 250, the median is 250. The mean is a simple calculation: $(150 + 180 + 190 + 230 + 250 + 250 + 280 + 300 + 340 + 380)/10 = 255$. The standard deviation is calculated just as easily:

$$\sqrt{\frac{697300 - \frac{2550^2}{10}}{9}} = 72.$$

(b) The Bright Idea Lighting Company's bulbs have a higher mean life.

(c) For the Bright Idea Lighting Company, we have

$$P(X > 350) = P\left(Z > \frac{350 - 262}{41}\right) = P(Z > 2.15) = 0.0158.$$

For The Electric Company,

$$P(X > 350) = P\left(Z > \frac{350 - 255}{72}\right) = P(Z > 1.32) = 0.0934.$$