Stat 151.003 (Kozdron) Fall 2006
Solutions to Practice Problems for Midterm #2

1.

(a) An approximate 95% confidence interval for the true daily parking revenues is

$$\left(\overline{X} - z_{0.025} \cdot \frac{S}{\sqrt{n}}\,,\, \overline{X} + z_{0.025} \cdot \frac{S}{\sqrt{n}}\right)$$

or

$$\left(126 - 1.96 \cdot \frac{15}{\sqrt{44}}\,,\, 126 + 1.96 \cdot \frac{15}{\sqrt{44}}\right)$$

or

$$(121.6, 130.4).$$

(b) Based on the sample mean obtained, we are 95% confident that the true mean lies between 121.6 and 130.4.

(c) Yes, the consultant could have been correct since the predicted average of \$130 per day falls within the 95% confidence inteveral.

2.

(a) Let X denote the IQ score of one randomly selected child from this population so that X is normal with mean 100 and standard deviation 16. Therefore,

$$P(80 < X < 120) = P\left(\frac{80-100}{16} < \frac{X-100}{16} < \frac{120-100}{16}\right) = P(-1.25 < Z < 1.25) = 0.7888$$

where the last equality followed from Table E.

(b) If $\overline{X}$ denotes the average IQ score for these five randomly selected children, then $\overline{X}$ is normal with mean 100 and standard deviation $16/\sqrt{5}$. Therefore,

$$P(80 < \overline{X} < 120) = P\left(\frac{80-100}{16/\sqrt{5}} < \frac{\overline{X}-100}{16/\sqrt{5}} < \frac{120-100}{16/\sqrt{5}}\right) = P(-2.80 < Z < 2.80) = 0.9948.$$

(c) From **(a)**, we conclude that the probability one child has a score less than 80 is $p = (1 - 0.7888)/2 = 0.1056$. Therefore, the probability that one of them will have an IQ score of 80 or less and four will have IQ scores higher than 80 is

$$\binom{5}{1} p(1-p)^4 = \frac{5!}{1!4!}(0.8944)^4(0.1056) = 0.3379.$$

3. An approximate 95% confidence interval for the delay time of Airline A is

$$\left(\overline{X} - t_{0.025,19} \cdot \frac{S}{\sqrt{n}}\,,\, \overline{X} + t_{0.025,19} \cdot \frac{S}{\sqrt{n}}\right)$$

or

$$\left(30 - 2.093 \cdot \frac{10}{\sqrt{20}}, \, 30 + 2.093 \cdot \frac{10}{\sqrt{20}}\right)$$

or

$$(25.3, 34.7).$$

An approximate 95% confidence interval for the delay time of Airline W is

$$\left(\overline{X} - t_{0.025,19} \cdot \frac{S}{\sqrt{n}}, \, \overline{X} + t_{0.025,19} \cdot \frac{S}{\sqrt{n}}\right)$$

or

$$\left(20 - 2.093 \cdot \frac{15}{\sqrt{20}}, \, 20 + 2.093 \cdot \frac{15}{\sqrt{20}}\right)$$

or

$$(13.0, 27.0).$$

Since the two 95% confidence intervals overlap, we are 95% confident that there is no significant difference in the true mean delay times for Airline A and Airline W. Therefore, if the decision to recommend an airline is made on the basis of the mean waiting time only, then both airlines are equivalent. Flip a coin to pick one! (Of course, other correct answers are valid. I chose to use a t-based confidence interval since the data are only approximately normally distributed, and there is a small sample size. If you want to argue that the population is exactly normal with a known standard deviation so that a z-based confidence is appropriate, that would be acceptable.)

4.

(a) The approximate distribution of the mean NOX emission level $\overline{X}$ for the company's cars is normal with mean 0.9 grams per kilometre and standard deviation $0.15/\sqrt{125}$ grams per kilometre.

(b) We seek L such that $P(\overline{X} > L) = 0.01$. Normalizing gives

$$P(\overline{X} > L) = P\left(\frac{\overline{X} - 0.9}{0.15/\sqrt{125}} > \frac{L - 0.9}{0.15/\sqrt{125}}\right) = P\left(Z > \frac{L - 0.9}{0.15/\sqrt{125}}\right) = 0.01.$$

From Table E we find $P(Z > 2.33) = 0.01$. This then gives

$$\frac{L - 0.9}{0.15/\sqrt{125}} = 2.33$$

so that $L = 0.9312$.

5. Let p denote the true proportion of spinach plants who are adversely affected by sulfur dioxide. *You then need to make a decision as to what is an appropriate rate of severe leaf damage is.* I believe that it is reasonable to assume that sulfur dioxide has no effect if at most 1/3 of spinach plants are affected. Therefore,

- H_0 : no effect: $p \leq 1/3$,
- H_1 : an adverse effect: $p > 1/3$.