Stat 151.003 Fall 2006 (Kozdron)
Solutions to Assignment #7

1. This is a very open-ended problem. A complete, reasonable solution contains the following elements. Let μ_1 denote the true average age of married males, and let μ_2 denote the true age of married females. The appropriate hypotheses are therefore $H_0 : \mu_1 - \mu_2 = 0$ vs. $H_1 : \mu_1 - \mu_2 > 0$. Since there are 40 couples, the appropriate test is the two-sample z-test. In order to use the z-test, the ages of the married men and married women should be approximately normally distributed. This can be checked by plotting a histogram of each of the 40 sampled males ages, and the 40 sampled females ages.

2. (a) Since there are less than 30 data points in each subpopulation, and since the population standard deviation is unknown, we will conduct a t-test. The t-test statistic is given by

$$t = \frac{\overline{X}_1 - \overline{X}_2}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_1^2}{n_1}}} = \frac{30.0 - 20.0}{\sqrt{\frac{77}{7} + \frac{35}{7}}} = 2.5.$$

There are $\min\{7-1, 7-1\} = 6$ degrees of freedom, and so from Table F, the rejection region is given by all those values of the t-statistic larger than 1.943. We therefore reject H_0 at the $\alpha = 0.05$ significance level.

(b) When we look at Table F we see that for a one-tailed test of $H_0 : \mu_1 - \mu_2 = 0$ vs. $H_1 : \mu_1 - \mu_2 > 0$ with 6 degrees of freedom, the critical value corresponding to $\alpha = 0.025$ is 2.447, and that the critical value corresponding to $\alpha = 0.01$ is 3.143. Thus, we conclude that

$$0.01 < P\text{-value} < 0.025.$$

3. Let μ_1 denote the true average salary for those with master's degrees, and let μ_2 denote the true average salary for those with bachelor's degrees. We are interested in testing $H_0 : \mu_1 - \mu_2 = 0$ vs. $H_1 : \mu_1 - \mu_2 > 0$. We use a z-test since there are greater than 30 data points in each sample. The z-test statistic is given by

$$z = \frac{\overline{X}_1 - \overline{X}_2}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_1^2}{n_1}}} = \frac{41\ 000 - 38\ 000}{\sqrt{\frac{2500^2}{60} + \frac{2000^2}{80}}} = 7.641.$$

The critical value for significance level $\alpha = 0.10$ for this z-test is given by 1.282. Since $7.641 > 1.282$, there is overwhelming evidence to conclude that individuals with a master's degree have higher starting salaries than those with just a bachelor's degree.

4. (a) Let p_1 denote the true proportion of African miners who died on the Gold Coast in 1936, and let p_2 denote the true proportion of European miners who died on the Gold Coast in 1936. Therefore, we are interested in testing $H_0 : p_1 - p - 2 = 0$ vs. $H_1 : p_1 - p_2 > 0$.

(b) The appropriate test statistic is

$$z = \frac{p_1 - p_2}{\sqrt{\overline{p}(1-\overline{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

where $\hat{p}_1 = \frac{223}{33\ 809}$, $\hat{p}_2 = \frac{7}{1541}$, and $\overline{p} = \frac{223+7}{33\ 809+1541}$. Substituting in gives

$$z = \frac{p_1 - p_2}{\sqrt{\overline{p}(1-\overline{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = 0.98.$$

(c) From Table E, the corresponding P-value is 0.1635.

(d) Since P-value$> \alpha = 0.05$, we do not reject H_0. That is, there is not good evidence to suggest that the proportion of African miners who died on the Gold Coast in 1936 is higher than the proportion of European miners who died on the Gold Coast in 1936.