Stat 151.003 Fall 2006 (Kozdron) Solutions to Assignment #6

1. Since there are less than 30 data points, and since the population standard deviation is unknown, we will conduct a *t*-test. We calculate the sample mean $\overline{X} = 5.5$ and the sample standard deviation S = 2.517. The *t*-test statistic is therefore given by

$$t = \frac{\overline{X} - \mu}{S/\sqrt{n}} = \frac{5.5 - 0}{2.517/\sqrt{9}} = 6.556.$$

There are n - 1 = 8 degrees of freedom, and so from Table F, the rejection region is given by all those values of the *t*-statistic larger than 3.355. We therefore reject H_0 and conclude that there is significant evidence at the $\alpha = 0.01$ level that there is a percent change in blood polyphenols among men who drink a half-bottle of wine per day.

2. (a) There are n - 1 = 9 degrees of freedom.

(b) When we look at Table F we see that for a one-tailed test of $H_0: \mu = 10$ v.s. $H_1: \mu < 10$ with 9 degrees of freedom, the critical value corresponding to $\alpha = 0.025$ is -2.262, and that the critical value corresponding to $\alpha = 0.05$ is -1.833. Thus, we conclude that

$$0.025 < P$$
-value < 0.05 .

3. The critical value for significance level $\alpha = 0.01$ for the *t*-test of $H_0: \mu = 0$ v.s. $H_1: \mu \neq 0$ with 19 degrees of freedom is given by 2.861 from Table F. Therefore, all values of the *t*-statistic which are smaller than -2.861 and larger than 2.861 are statistically significant at this level.

4. (a) We compute $\overline{X} = 1.02$.

(b) Although there are only 10 data points, they are assumed to come from a normal population with known standard deviation (namely $\sigma = 1$). Therefore, we will conduct a z-test. Let μ denote the true average sweetness loss for this cola so that the appropriate hypotheses are $H_0: \mu = 0$ vs. $H_1: \mu > 0$. Since $\overline{X} = 1.02$, the z-test statistic is given by

$$z = \frac{X - \mu}{\sigma / \sqrt{n}} = \frac{1.02 - 0}{1 / \sqrt{10}} = 3.26.$$

From Table E or Table F, the rejection region is given by all those values of the z-statistic larger than 2.326. We therefore reject H_0 and conclude that there is significant evidence at the $\alpha = 0.01$ level that there is a loss of sweetness for this cola.