Math 026L.04 Spring 2002
Test #4

Name: ______________________________

Read all of the following information before starting the test:

- Be sure that this test has **7** numbered pages.
- There are **four** problems on this test worth a total of **50** points. You must solve problems **1** and **2**. You must then choose either problem **3** or problem **4** to solve. Indicate below which problem you have chosen.

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  You may choose to solve the other problem which is worth 4 bonus points.
- The last page is for your scrap work and may be detached from the test booklet.
- Calculators are permitted, but no other aids are allowed. When you do use your calculator, sketch all relevant graphs and write down all relevant mathematics.
- Show all work neatly and in order, and clearly indicate your final answers.
- Answers must be justified whenever possible in order to earn full credit. ***No credit will be given for unsupported answers, even if your final answer is correct.***
- Please keep your written answers succinct. Points will be deducted for incoherent, incorrect and/or irrelevant statements.
- Good luck!

Problem	1	2	3	4	Total
Score					

1. (*24 points*) Consider the following game. A player rolls a fair die. If the die's upmost face is either a 1 or a 6, then the player flips two fair coins and wins \$$X$, where X is the number of heads flipped. However, if the die's upmost face is either a 2, 3, 4, or 5, then the player flips only one coin and instead wins \$$X$, where X is the number of heads on this single flip.

(a) What are all possible values for X, the player's winnings?

(b) Compute the probability mass density of X. Display your answer in a table.

(Continued)

(c) Determine what a fair price to pay to play is by computing $\mathbb{E}(X)$.

2. (*16 points*) Studies have shown that if a glass of milk is left out in the sun it will go sour within 2 hours. It has also been shown that the time taken to go sour is distributed according to the density function

$$p(t) = \frac{3}{8}t^2$$

for $0 \leq t \leq 2$.

(a) Carefully verify that $p(t)$ is, in fact, a density function.

(b) Compute $P(t)$, the cumulative distribution function associated with this density.

(Continued)

(c) Compute the median of this density. That is, for what value of a does $\displaystyle\int_0^a p(t)\,dt = \frac{1}{2}$.

(d) Compute the mean of this density. That is, compute $\displaystyle\int_0^2 t\,p(t)\,dt$.

3. (*10 points*) On Test #3 you investigated the Cauchy density function. The Cauchy density for $0 \leq t < \infty$ is given by

$$p(t) = \frac{2}{\pi} \frac{1}{1+t^2}.$$

It is also true that $\displaystyle\int_0^\infty p(t)\, dt = 1$.

However, the Cauchy density is unique in that it does not have a finite mean. Carefully show that the mean

$$\int_0^\infty t\, p(t)\, dt$$

is infinite.

4. (*10 points*) Suppose that the scores on a certain calculus test were distributed according to the **distribution** function

$$P(t) = \frac{2}{\pi}\arcsin(\sqrt{t})$$

where $0 < t < 1$ represents the score as a decimal.

(a) What is the density function $p(t)$ associated with $P(t)$?

(b) What fraction of students received a score less than 0.60?

(Hint: This part cannot be solved with your answer from ***(a)****. Think about what information the distribution function gives.)*

Scrap Page

(*You may carefully remove this page from the test booklet.*)