Math 026L.04 Spring 2002
Test #1 Solutions

1.

(a) $\frac{\pi}{3}$

(b) $-\dfrac{\sqrt{3}}{2}$

(c) $\dfrac{1}{\sqrt{1+x^2}}$

2.

Let $\theta = \tan^{-1} x$. Then $\tan\theta = x$. Differentiating both sides with respect to x gives $\frac{d}{dx}\tan\theta = \frac{d}{dx}x$, or

$$\frac{1}{\cos^2\theta}\frac{d\theta}{dx} = 1.$$

Hence,

$$\frac{d\theta}{dx} = \cos^2\theta = \cos^2(\tan^{-1} x) = \left(\frac{1}{\sqrt{1+x^2}}\right)^2 = \frac{1}{1+x^2}$$

by Problem 1 **(c)**.

3.

(a) $y' = \dfrac{\sin x}{\cos^2 x}$

(b) $y' = 2\cos(\cos x)\cdot\sin(\cos x)\cdot\sin x$

(c) $y' = \dfrac{-1}{\sqrt{1-(\sin^{-1}x)^2}}\cdot\dfrac{1}{\sqrt{1-x^2}}$

4.

$x = 718.89$ and $y = 541.72$

5.

(a) On the given domain, $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, your sine curve should complete exactly one period.

(b) $\text{Range}(y) = [1, 7]$

(c) y is not invertible since it fails the Horizontal Line Test. For example, $y(-\pi/2) = y(0) = y(\pi/2) = 4$.

(d) N/A

(e) $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$

6.

(a) $v(t) = -A\omega \sin(\omega t + k)$

(b) $a(t) = -A\omega^2 \cos(\omega t + k)$

(c) Since $s''(t) = -A\omega^2 \cos(\omega t + k)$, and $-\omega^2 s(t) = -\omega^2 \cdot A \cos(\omega t + k)$, and these two are equal, $s(t)$ is a solution to the differential equation $s''(t) = -\omega^2 s(t)$.

(d) In this case, $v(t) = -\sin(\frac{1}{2}t + \pi)$ so that $v(t) = 0$ when $\sin(\frac{1}{2}t + \pi) = 0$.

But $\sin x = 0$ whenever x is a multiple of π. Thus we need $\frac{1}{2}t + \pi$ to be a multiple of π.

This happens only when $t = 0, \pm 2\pi, \pm 4\pi, \pm 6\pi, \ldots$.

So, in particular, three exact values of t are $t = 0, \pi, -\pi$.

7.

(a) To solve this problem start by plotting the 5 given points. You will notice that the three points $(0, 20)$, $(6, 35)$, and $(12, 50)$ all line in a perfectly straight line.

This gives the linear component of the stock's price. The slope of the line through these points is $m = \frac{50-20}{12-0} = \frac{5}{2}$ and since the line passes through $(0, 20)$, the y-intercept is $b = 20$.

Now, the tricky part.

If we look at $f(t)-$ the linear part, we are left with only the cyclical part. That is,

$$f(t) - mt - b = A \sin \frac{\pi t}{6}, \text{ or}$$

$$f(t) - \frac{5}{2}t - 20 = A \sin \frac{\pi t}{6}.$$

Plotting each of the 5 points *with the linear part subtracted*, gives us a curve which goes through the 5 points $(0, 0)$, $(3, 10)$, $(6, 0)$, $(9, -10)$, and $(12, 0)$.

Thus these points fall exactly on the sine curve with amplitude $A = 10$.

$$\therefore f(t) = \frac{5}{2}t + 20 + 10 \sin \frac{\pi t}{6}$$

(b) Notice that the five original data points lie exactly on the curve $f(t)$.

(c) Looking at the graph of $f(t)$ it appears that the stock is actually losing value between May and September.