Math 026L.04 Spring 2002
Test #1

Name: ______________________________

Read all of the following information before starting the test:

- Be sure that this test has **12** numbered pages.
- There are **7** problems on this test, plus some bonus problems at the end, worth a total of **100** points.
- The last page is for your scrap work and may be detached from the test booklet.
- Calculators are permitted, but no other aids are allowed. When you do use your calculator, sketch all relevant graphs and write down all relevant mathematics.
- Show all work neatly and in order, and clearly indicate your final answers.
- Answers must be justified whenever possible in order to earn full credit. ***No credit will be given for unsupported answers, even if your final answer is correct.***
- Please keep your written answers succinct. Points will be deducted for incoherent, incorrect and/or irrelevant statements.
- Good luck!

Problem	1	2	3	4	5	6	7	B	S	Total
Score										

1. (*12 points*)
Find the **exact** value of each of the following expressions.

(a) $\sin^{-1} \dfrac{\sqrt{3}}{2}$

(b) $\cos \dfrac{-5\pi}{6}$

(c) $\cos\left(\tan^{-1} x\right)$

2. (*10 points*)

Using the chain rule, verify that the derivative of $\tan^{-1}(x)$ is $\frac{1}{1+x^2}$. In other words, verify that

$$\frac{d}{dx}\tan^{-1}(x) = \frac{1}{1+x^2}.$$

3. (*12 points*)

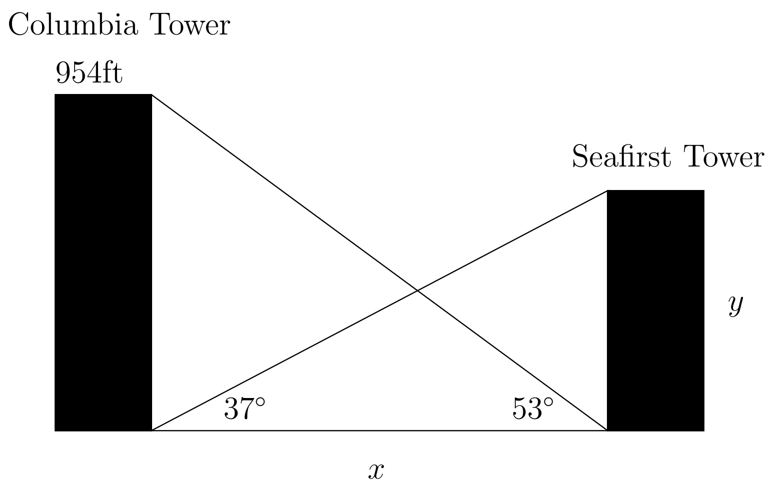
For each of the functions given below, compute $\frac{dy}{dx}$.

(a) $y = \frac{\tan(x)}{\sin(x)}$

(b) $y = \cos^2(\cos x)$

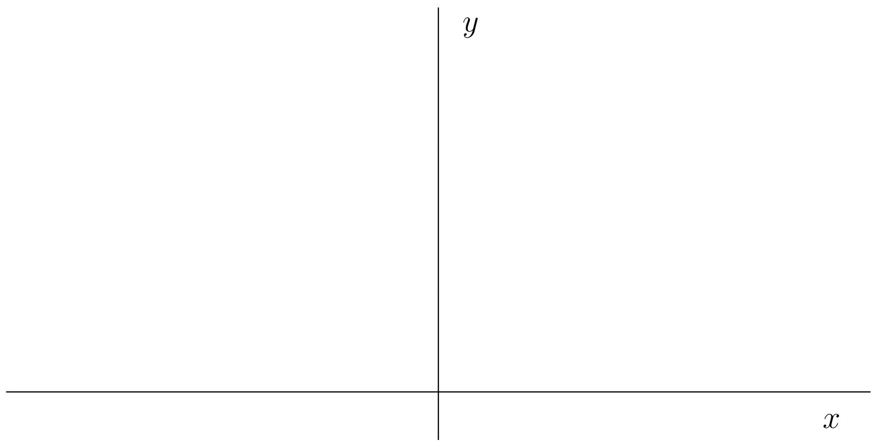
(c) $y = \cos^{-1}\left(\sin^{-1}(x)\right)$

4. (*10 points*)
Suppose that the Columbia Tower in Seattle is 954 ft tall. The angle the top of each building makes with the base of the other building is shown in the figure below. Determine both y, the height of the Seafirst Tower, and x, the distance between the two towers.



5. (*18 points*)
Consider the function $y = 3\sin\left(2\left(x - \frac{\pi}{2}\right)\right) + 4$ *restricted* to the domain $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

(a) On the axes below sketch the graph of y. Be sure to label points on your axes.



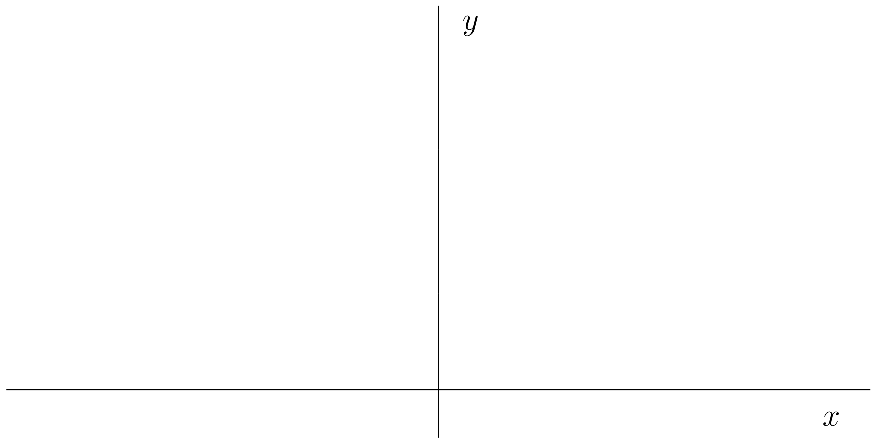
(b) What is the range of y?

(c) Is y an invertible function? Why or why not?

(continued)

Answer either **(d)** or **(e)** based on your response to **(c)**.

(d) If y is invertible, sketch its inverse, y^{-1} on the axes below.



(e) If y is not invertible, what is the largest domain containing 0 on which it is invertible?

6. (*18 points*)
If the equation of motion of a particle is given by $s(t) = A\cos(\omega t + k)$, where A, ω, and k are non-zero constants, then the particle is said to undergo *simple harmonic motion.*

(a) Find $v(t)$, the velocity of the particle at time t.

(b) Find $a(t)$, the acceleration of the particle at time t.

(c) Verify that $s(t)$ is a solution to the differential equation $\dfrac{d^2s(t)}{dt^2} = -\omega^2 s(t)$.

(continued)

For **(d)** suppose that $A = 2$, $\omega = \frac{1}{2}$, and $k = \pi$.

(d) Find two exact values of t for which the velocity is 0.

7. (*20 points*)
Benoit wants to develop a mathematical model for predicting the value of a certain stock on the New York Stock Exchange. He has made observations from the past behaviour of the stock:

(1) its value seems to have a cyclical component which increases for the first three months of each year, falls for the next six, and then rises for the last three;

(2) inflation adds a linear component to the stock's price.

For these reasons Benoit is seeking a model of the form

$$f(t) = mt + b + A\sin\frac{\pi t}{6},$$

where t represents the time in months after January 1, 1990, and m, b, and A are constants.

He has the following data:

Date	Jan 1, 1990	Apr 1, 1990	Jul 1, 1990	Oct 1, 1990	Jan 1, 1991
Stock Price	$20.00	$37.50	$35.00	$32.50	$50.00

(a) Find values of m, b, and A so that f fits the data.

(continued)

(b) Sketch both the data points and your function $f(t)$ from **(a)** on the same set of axes.

(c) During what period each year is this stock actually losing value?

Bonus Question (*2 bonus points*)

Use the chain rule to show that if θ is measured in degrees, then

$$\frac{d}{d\theta}\cos\theta = -\frac{\pi}{180}\sin\theta.$$

Survey Question (*1 bonus point*)

What did you think of this test? Was it what you were expecting?

Scrap Page

(*You may carefully remove this page from the test booklet.*)