Logistic Growth

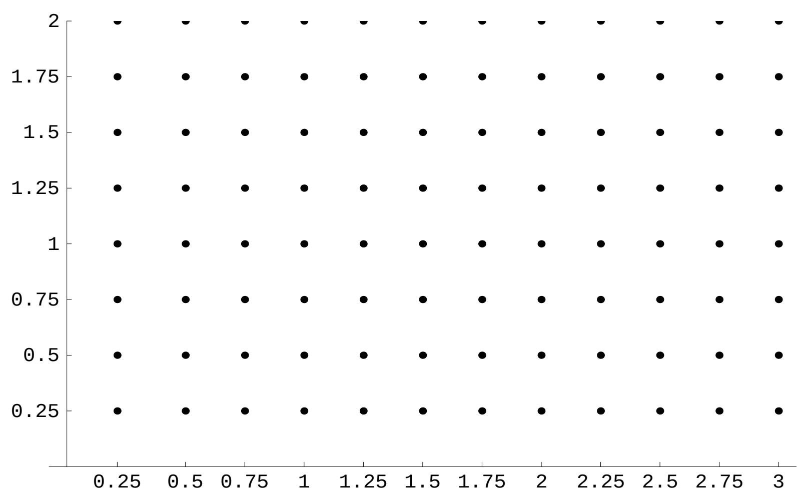
Purpose: The purpose of this lab is to collect and examine data from a simulation of the spread of a rumor within a fixed population and to examine a differential equation and its solution that provide a mathematical model for the situation.

Part I: Data Collection

Under the direction of your instructor or lab TA collect and record the data for the simulation of the spread of a rumor.

Part II: A Simple Logistic Equation: $\dfrac{dy}{dt} = 2y(1-y)$

1. Sketch below the slope field for $\dfrac{dy}{dt} = 2y(1-y)$ and sketch two solutions.



2. Use Euler's method with $\Delta t = 0.25$ and with the initial condition $(0, 0.1)$ to graph an estimate of the solution of $\dfrac{dy}{dt} = 2y(1-y)$ between $t = 0$ and $t = 3$.

Part III: The General Logistic Equation: $\dfrac{dy}{dt} = ky(M - y)$

1. Approximately what is $\frac{dy}{dt}$ when y is close to M? If y_0 is very small relative to M then while t is small, $M - y$ will be approximately constant. During that period of time what will the shape of the graph of y look like?

2. Show that $y = \dfrac{My_0}{(M - y_0)e^{-Mkt} + y_0}$ is a solution to $\dfrac{dy}{dt} = ky(M - y)$ where $y = y_0$ when $t = 0$ in the particular case when $k = 2$, $M = 1$, and $y_0 = 0.1$.

3. Graph the solution to $\frac{dy}{dt} = ky(M - y)$ where $y = y_0$ when $t = 0$ in the following particular cases.

(a) $k = 2$, $M = 1$, and $y_0 = 0.1$

(b) $k = 1$, $M = 100$, and $y_0 = 2$

4. Is your answer to Part III question 1 consistent with your answer to Part III question 2?

Part IV: The Data and its Relationship to the Logistic Equation

1. Think about the spread of the rumor in the simulation and suppose that the size of the class had been very large (say the entire population of east campus). At the beginning and for some time after that it would not be very likely that anyone you told the rumor to would have already heard it. So if two people start the rumor, the rumor would spread at first like 2, 4, 8, etc. How would you describe such growth and is that consistent with the properties of logistic growth described above?

2. As the rumor spreads eventually the whole class has heard it. How is this consistent with the properties of logistic growth described above?

3. Graph the data you collected. Does it have the same basic shape of the solution to the logistic equation?

4. If the data collected can be modeled with the logistic equation, what must M be? Use the data to estimate k. Write down a formula that approximates the data. How well does it approximate the data?