Math 26L.04 Spring 2002
Solutions: Extra Review Problems for Test #3

1. Look over Assignment #6, Assignment #7, Assignment #8, Quiz #6, Quiz #7, Quiz #8, and the extra problems solved in the class notes.

2. Done in class.

3. Done in class.

4. Separate into $y \sin y \dfrac{dy}{dx} = \dfrac{\ln x}{x}$. Now antidifferentiate both sides separately.

Thus,

$$\int \frac{\ln x}{x}\, dx = \int u\, du = \frac{1}{2}u^2 + A = \frac{1}{2}(\ln x)^2 + A,$$

and

$$\int y \sin y\, dy = -y\cos y + \sin y + B.$$

Combining these gives $-y\cos y + \sin y = \frac{1}{2}(\ln x)^2 + C$. Substituting in $y(1) = \pi$ gives $C = \pi$. Therefore,

$$-y\cos y + \sin y = \frac{1}{2}(\ln x)^2 + \pi.$$

5.

(a) Let $f(x) = \frac{1}{1+a^x}$ and $\Delta x = \frac{4-0}{4} = 1$.

LHS$= 1 \cdot f(0) + 1 \cdot f(1) + 1 \cdot f(2) + 1 \cdot f(3) = \dfrac{1}{1+a^0} + \dfrac{1}{1+a^1} + \dfrac{1}{1+a^2} + \dfrac{1}{1+a^3}$

(b) Again let $f(x) = \frac{1}{1+a^x}$ and $\Delta x = \frac{4-0}{4} = 1$.

RHS$= 1 \cdot f(1) + 1 \cdot f(2) + 1 \cdot f(3) + 1 \cdot f(4) = \dfrac{1}{1+a^1} + \dfrac{1}{1+a^2} + \dfrac{1}{1+a^3} + \dfrac{1}{1+a^4}$

(c) If $a \geq 1$, then $\dfrac{1}{1+a^x}$ is strictly decreasing. Thus, LHS $> \displaystyle\int_0^4 \frac{1}{1+a^x}\, dx >$ RHS.

(d) LHS$= \displaystyle\frac{1}{25}\sum_{k=0}^{99} \frac{1}{1+2^{k/25}} \approx .9213787$

(e) RHS$= \displaystyle\frac{1}{25}\sum_{k=1}^{100} \frac{1}{1+2^{k/25}} \approx .9037316$

6. Done in class.

7. We begin by computing $\int x^{3-1}e^{-x}\,dx$. Use parts twice.

Let $u = x^2$, $dv = e^{-x}$. Then,

$$\int x^2 e^{-x}\,dx = -x^2e^{-x} + 2\int xe^{-x}\,dx.$$

Use parts again with $u = x$, $dv = e^{-x}$. Then,

$$-x^2e^{-x} + 2\int xe^{-x}\,dx = -x^2e^{-x} + 2\left(-xe^{-x} + \int e^{-x}\,dx\right) = -x^2e^{-x} - 2xe^{-x} - 2e^{-x} + C.$$

Now, by definition, $\int_0^\infty x^{p-1}e^{-x}\,dx = \lim_{b\to\infty}\int_0^b x^{p-1}e^{-x}\,dx$.

But

$$\int_0^b x^{p-1}e^{-x}\,dx = \left[-x^2e^{-x} - 2xe^{-x} - 2e^{-x} + C\right]_0^b = (-b^2e^{-b} - 2be^{-b} - 2e^{-b}) - (-2).$$

Finally,

$$\lim_{b\to\infty}(-b^2e^{-b} - 2be^{-b} - 2e^{-b}) - (-2) = 2.$$

Thus,

$$G = \frac{1}{2}.$$

8. Done in class.